Lecture 13

Linear programming :
Computational Procedure of Dual Simplex Method

13.1 Introduction

Any LPP for which it is possible to find infeasible but better than optimal initial basic solution can be solved by using dual simplex method. Such a situation can be recognized by first expressing the constraints in '≤' form and the objective function in the maximization form. After adding slack variables, if any right hand side element is negative and the optimality condition is satisfied then the problem can be solved by dual simplex method.

Negative element on the right hand side suggests that the corresponding slack variable is negative. This means that the problem starts with optimal but infeasible basic solution and we proceed towards its feasibility.

The dual simplex method is similar to the standard simplex method except that in the latter the starting initial basic solution is feasible but not optimum while in the former it is infeasible but optimum or better than optimum. The dual simplex method works towards feasibility while simplex method works towards optimality.

13.2 Computational Procedure of Dual Simplex Method

The iterative procedure is as follows

Step 1 - First convert the minimization LPP into maximization form, if it is given in the minimization form.

Step 2 - Convert the '≥' type inequalities of given LPP, if any, into those of '≤' type by multiplying the corresponding constraints by -1.

Step 3 – Introduce slack variables in the constraints of the given problem and obtain an initial basic solution.

Step 4 – Test the nature of Δ_j in the starting table

- If all Δ_j and X_B are non-negative, then an optimum basic feasible solution has been attained.
- If all Δ_j are non-negative and at least one basic variable X_B is negative, then go to step 5.
- If at least Δ_j one is negative, the method is not appropriate.

Step 5 – Select the most negative X_B. The corresponding basis vector then leaves the basis set B. Let X_r be the most negative basic variable.

Step 6 – Test the nature of X_r

- If all X_r are non-negative, then there does not exist any feasible solution to the given problem.
- If at least one X_r is negative, then compute Max (Δ_j / X_r) and determine the least negative for incoming vector.

Step 7 – Test the new iterated dual simplex table for optimality.

Repeat the entire procedure until either an optimum feasible solution has been attained in a finite number of steps.

13.3 Worked Examples

Example 1

Minimize $Z = 2x_1 + x_2$

Subject to

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

and $x_1 \geq 0, x_2 \geq 0$

Solution

Step 1 – Rewrite the given problem in the form

Maximize $Z' = -2x_1 - x_2$

Subject to

$-3x_1 - x_2 \leq -3$

$-4x_1 - 3x_2 \leq -6$

$-x_1 - 2x_2 \leq -3$

$x_1, x_2 \geq 0$

Step 2 – Adding slack variables to each constraint

Maximize $Z' = -2x_1 - x_2$

Subject to

$-3x_1 - x_2 + s_1 = -3$

$-4x_1 - 3x_2 + s_2 = -6$

$-x_1 - 2x_2 + s_3 = -3$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Step 3 – Construct the simplex table

	$C_j \rightarrow$		-2	-1	0	0	0	
Basic variables	C_B	X_B	X_1	X_2	S_1	S_2	S_3	
s_1	0	-3	-3	-1	1	0	0	
s_2	0	-6	-4	$\boxed{-3}$	0	1	0	$\rightarrow$ outgoing
s_3	0	-3	-1	-2	0	0	1	
	$Z' = 0$		2	$\uparrow$ 1	0	0	0	$\leftarrow \Delta_j$

Step 4 – To find the leaving vector
Min (-3, -6, -3) = -6. Hence s_2 is outgoing vector

Step 5 – To find the incoming vector
Max (Δ_1 / x_{21}, Δ_2 / x_{22}) = (2/-4, 1/-3) = -1/3. So x_2 is incoming vector

Step 6 –The key element is -3. Proceed to next iteration

	$C_j \rightarrow$		-2	-1	0	0	0	
Basic variables	C_B	X_B	X_1	X_2	S_1	S_2	S_3	
s_1	0	-1	$\boxed{-5/3}$	0	1	-1/3	0	$\rightarrow$ outgoing
x_2	-1	2	4/3	1	0	-1/3	0	
s_3	0	1	5/3	0	0	-2/3	1	
	$Z' = -2$		$\uparrow$ 2/3	0	0	1/3	0	$\leftarrow \Delta_j$

Step 7 – To find the leaving vector
Min (-1, 2, 1) = -1. Hence s_1 is outgoing vector

Step 8 – To find the incoming vector
Max (Δ_1 / x_{11}, Δ_4 / x_{14}) = (-2/5, -1) = -2/5. So x_1 is incoming vector

Step 9 –The key element is -5/3. Proceed to next iteration

	$C_j \rightarrow$		-2	-1	0	0	0	
Basic variables	C_B	X_B	X_1	X_2	S_1	S_2	S_3	
x_1	-2	3/5	1	0	-3/5	1/5	0	
x_2	-1	6/5	0	1	4/5	-3/5	0	
s_3	0	0	0	0	1	-1	1	
	$Z' = -12/5$		0	0	2/5	1/5	0	$\leftarrow \Delta_j$

Step 10 – $\Delta_j \geq 0$ and $X_B \geq 0$, therefore the optimal solution is Max $Z' = -12/5$, $Z = 12/5$, and $x_1 = 3/5$, $x_2 = 6/5$

Example 2

Minimize $Z = 3x_1 + x_2$
Subject to
$x_1 + x_2 \geq 1$
$2x_1 + 3x_2 \geq 2$
and $x_1 \geq 0$, $x_2 \geq 0$

Solution

Maximize $Z' = -3x_1 - x_2$
Subject to
$-x_1 - x_2 \leq -1$
$-2x_1 - 3x_2 \leq -2$
$x_1, x_2 \geq 0$

SLPP
Maximize $Z' = -3x_1 - x_2$
Subject to
$-x_1 - x_2 + s_1 = -1$
$-2x_1 - 3x_2 + s_2 = -2$
$x_1, x_2, s_1, s_2 \geq 0$

	$C_j \rightarrow$		-3	-1	0	0	
Basic variables	C_B	X_B	X_1	X_2	S_1	S_2	
s_1	0	-1	-1	-1	1	0	
s_2	0	-2	-2	$\boxed{-3}$	0	1	$\rightarrow$
	$Z' = 0$		3	$\uparrow$ 1	0	0	$\leftarrow \Delta_j$
s_1	0	-1/3	-1/3	0	1	$\boxed{-1/3}$	$\rightarrow$
x_2	-1	2/3	2/3	1	0	-1/3	
	$Z' = -2/3$		7/3	0	0	$\uparrow$ 1/3	$\leftarrow \Delta_j$
s_2	0	1	1	0	-3	1	
x_2	-1	1	1	1	-1	0	
	$Z' = -1$		2	0	1	0	$\leftarrow \Delta_j$

$\Delta_j \geq 0$ and $X_B \geq 0$, therefore the optimal solution is Max $Z' = -1$, $Z = 1$, and $x_1 = 0$, $x_2 = 1$