Description of The Algorithm

Dijkstra's algorithm works by solving the sub-problem k, which computes the shortest path **from the source to vertices among the k closest vertices to the source**. For the dijkstra's algorithm to work it should be **directed- weighted** graph and the **edges should be non-negative**. **If the edges are negative then the actual shortest path cannot be obtained.**

General Description

Suppose we want to find a shortest path from a given node **s** to other nodes in a network (one-to-all shortest path problem)

- Dijkstra's algorithm solves such a problem
- It finds the shortest path from a given node **s** to all other nodes in the network
- **Node s is called a starting node or an initial node**
- How is the algorithm achieving this?
- **Dijkstra's algorithm starts by assigning some initial values for the distances from node s and to every other node in the network**
- It operates in steps, where at each step the algorithm improves the distance values.
- At each step, the shortest distance from node **s** to another node is determined

Formal Description

The algorithm characterizes each node by its state.**The state of a node consists of two features**:

➢ **Distance value and status label**

- **Distance value** of a node is a scalar representing an estimate of the its distance from node **s**.
- **Status label** is an attribute specifying whether the distance value of **a** node is equal to the shortest distance to node **s** or not.
- The status label of **a** node is **Permanent** if its distance value is equal to the shortest distance from node **s**
- Otherwise, the status label of **a** node is **Temporary**

The algorithm maintains and step-by-step updates the states of the nodes. At each step one node is designated as current

Algorithm Steps

Step 1. Initialization

- Assign the **zero** distance value to node **s**, **and label it as Permanent**. [The state of node **s** is (0, p)]
- Assign to every node **a** distance value of ∞ and label them as **Temporary**. [The state of every other node is (∞, t)]
- Designate the node **s** as the **current node**

Step 2. Distance Value Update and Current Node Designation Update

Let **i** be the index of the **current node**.

(1) Find the set **J** of nodes with **temporary** labels that can be reached from the current node **i** by a link **(i, j)**. Update the distance values of these nodes.

- For each **j ∈ J**, the distance value $\mathbf{d_j}$ of node **j** is updated as follows

  new $\mathbf{d_j = \min\{d_j, d_i + c_{ij}\}}$

where $\mathbf{c_{ij}}$ is the cost of link **(i, j)**, as given in the network problem.

(2) Determine a node j that has the smallest distance value $\mathbf{d_j}$ among all nodes **j∈ J**, find **j*** such that

$$\min_{j \in J} d_j = d_{j^*}$$

(3) Change the label of node **j*** to **permanent** and designate this node as the **current node**.

Step 3. Termination Criterion

If all nodes that can be reached from node **s** have been **permanently** labeled, then stop - we are done.

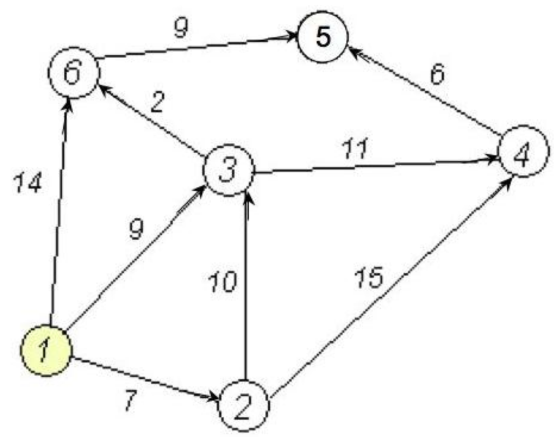
If we cannot reach any **temporary labeled** node from the current node, then all the temporary labels become permanent - we are done.

Otherwise, go to Step 2.

Dijkstra's Algorithm - Pseudocode

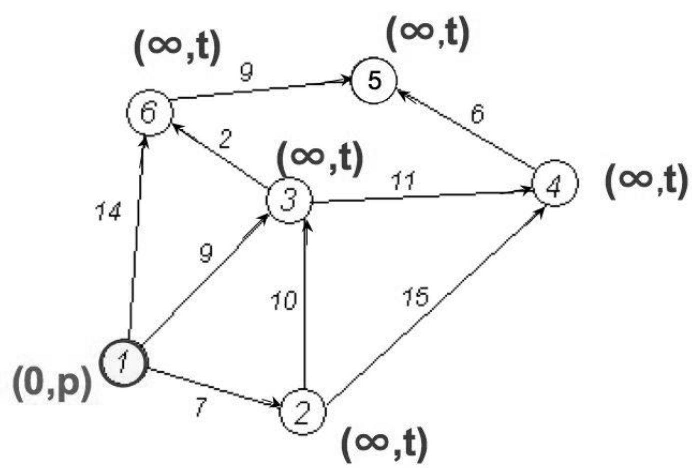
```
dist[s] ←0                               (distance to source vertex is zero)
for  all v ∈ V–{s}
       do  dist[v] ←∞              (set all other distances to infinity)
S←∅                              (S, the set of visited vertices is initially empty)
Q←V                                      (Q, the queue initially contains all vertices)
while Q ≠ ∅                      (while the queue is not empty)
do   u ← mindistance(Q,dist)          (select the element of Q with the min. distance)
     S←S ∪ {u}                        (add u to list of visited vertices)
      for all v ∈  neighbors[u]
            do  if   dist[v] > dist[u] + w(u, v)           (if new shortest path found)
                     then      d[v] ←d[u] + w(u, v)        (set new value of shortest path)
                    (if desired, add traceback code)
return dist
```

Example: We want to find the shortest path from **node 1** to the all the other nodes in the network using **Dijkstra's algorithm**



Step 1- Initialization

- Node **1** is designated as the **current node**
- **The state of node 1 is (0, p)**
- **Every other node has state (∞, t)**

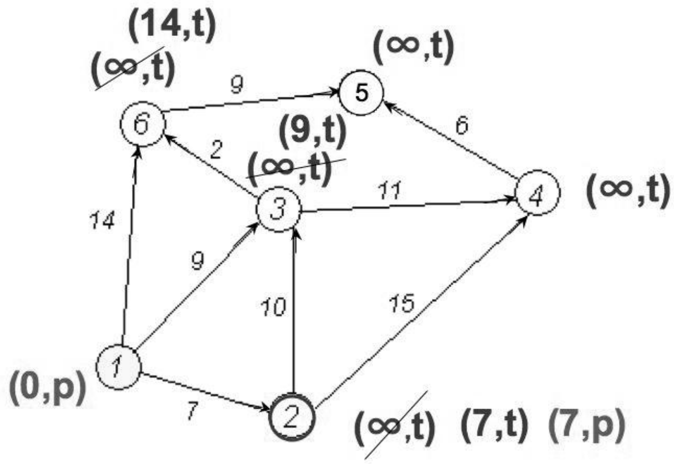


Step 2

Nodes 2, 3,and 6 can be reached from the current node 1

- **Update distance values for these nodes**

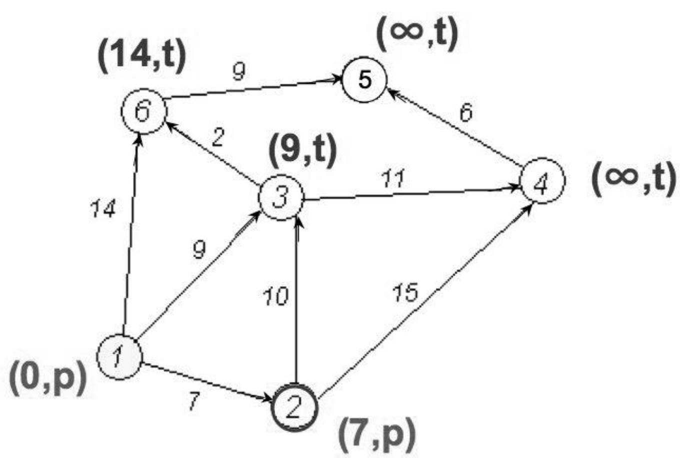
d2 = min{∞, 0+7} = 7
d3 = min{∞, 0+9} = 9
d6 = min{∞, 0+14} = 14



➢ Now, among the nodes **2, 3, and 6, node 2 has the smallest distance value**

➢ The **status label of node 2 changes to permanent**, so its state is (7, p), while the status of 3 and 6 remains temporary

➢ **Node 2 becomes the current node**



Step 3

Another Implementation of Step 2

- **Nodes 3 and 4 can be reached from the current node 2**
- Update distance values for these nodes

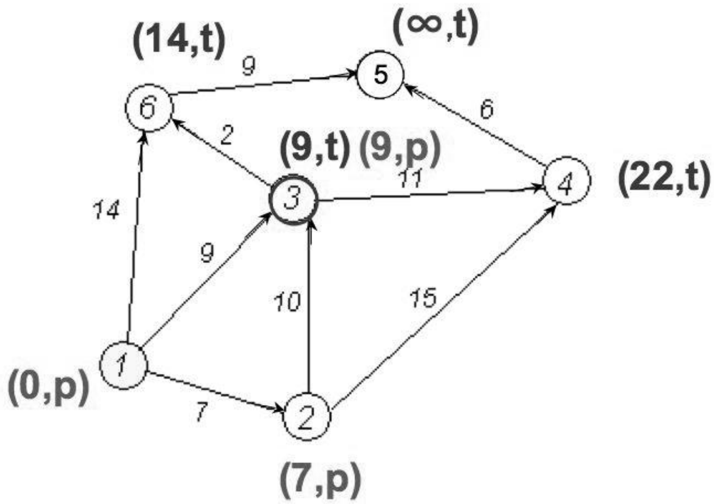
d3 = min{9, 7+10} = 9

d4 = min{∞, 7+15} = 22

- **Now, between the nodes 3 and 4 node 3 has the smallest distance value**
- The status label of **node 3 changes to permanent**, while the status of 4 remains temporary

- **Node 3 becomes the current node**

We are not done (Step 3 fails), **so we perform another Step 2**



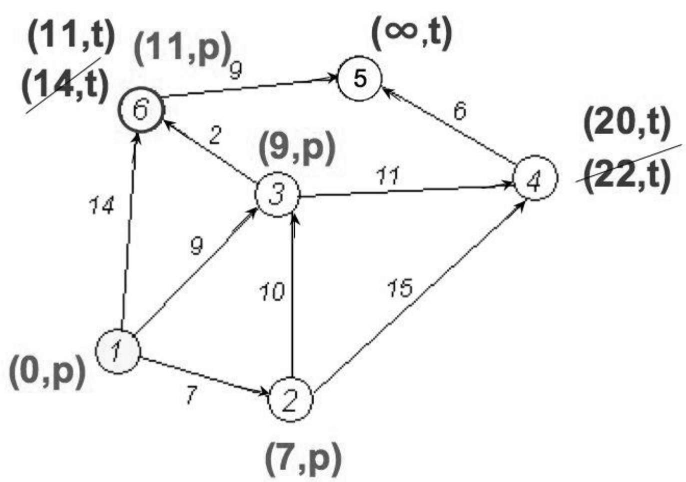
Another Step 2

- **Nodes 6 and 4 can be reached from the current node 3**
- Update distance values for them

d4 = min{22, 9+11} = 20

d6 = min{14, 9+2} = 11

- **Now, between the nodes 6 and 4 node 6 has the smallest distance value**
- The status label of **node 6 changes to permanent**, while the status of 4 remains temporary
- Node 6 becomes the current node we are not done (Step 3 fails), so we perform another Step 2



Another Step 2

- **Node 5 can be reached from the current node 6**
- Update distance value for node 5

$d5 = \min\{\infty, 11+9\} = 20$

- Now, **node 5 is the only candidate, so its status changes to permanent**
- **Node 5 becomes the current node**

From node 5 we cannot reach any other node. Hence, **node 4 gets permanently labeled and we are done.**