Lecture 16

Transportation problem : Methods for Initial Basic Feasible Solution (Vogal's Approximation method)

3 - Vogel's Approximation Method (Unit Cost Penalty Method)

Step1

For each row of the table, identify the **smallest** and the **next to smallest cost**. Determine the difference between them for each row. These are called **penalties**. Put them aside by enclosing them in the parenthesis against the respective rows. Similarly compute penalties for each column.

Step 2

Identify the row or column with the largest penalty. If a tie occurs then use an arbitrary choice. Let the largest penalty corresponding to the i^{th} row have the cost c_{ij}. Allocate the largest possible amount $x_{ij} = \min(a_i, b_j)$ in the cell (i, j) and cross out either i^{th} row or j^{th} column in the usual manner.

Step 3

Again compute the row and column penalties for the reduced table and then go to step 2. Repeat the procedure until all the requirements are satisfied.

Find the initial basic feasible solution using vogel's approximation method

1.

	W_1	W_2	W_3	W_4	Availability
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	

Solution

	W_1	W_2	W_3	W_4	Availability	Penalty
F_1	19	30	50	10	7	19-10=9
F_2	70	30	40	60	9	40-30=10
F_3	40	8	70	20	18	20-8=12
Requirement	5	8	7	14		
Penalty	40-19=21	30-8=22	50-40=10	20-10=10		

	W_1	W_2	W_3	W_4	Availability	Penalty
F_1	(19)	(30)	(50)	(10)	7	9
F_2	(70)	(30)	(40)	(60)	9	10
F_3	(40)	**8**(8)	(70)	(20)	18/10	12
Requirement	5	8/0	7	14		
Penalty	21	22	10	10		

	W_1	W_2	W_3	W_4	Availability	Penalty
F_1	**5**(19)	(30)	(50)	(10)	7/2	9
F_2	(70)	(30)	(40)	(60)	9	20
F_3	(40)	**8**(8)	(70)	(20)	18/10	20
Requirement	5/0	X	7	14		
Penalty	21	X	10	10		

	W_1	W_2	W_3	W_4	Availability	Penalty
F_1	**5**(19)	(30)	(50)	(10)	7/2	40
F_2	(70)	(30)	(40)	(60)	9	20
F_3	(40)	**8**(8)	(70)	**10**(20)	18/10/0	50
Requirement	X	X	7	14/4		
Penalty	X	X	10	10		

	W_1	W_2	W_3	W_4	Availability	Penalty
F_1	**5**(19)	(30)	(50)	**2**(10)	7/2/0	40
F_2	(70)	(30)	(40)	(60)	9	20
F_3	(40)	**8**(8)	(70)	**10**(20)	X	X
Requirement	X	X	7	14/4/2		
Penalty	X	X	10	50		

	W_1	W_2	W_3	W_4	Availability	Penalty
F_1	**5**(19)	(30)	(50)	**2**(10)	X	X
F_2	(70)	(30)	**7**(40)	**2**(60)	X	X
F_3	(40)	**8**(8)	(70)	**10**(20)	X	X
Requirement	X	X	X	X		
Penalty	X	X	X	X		

Initial Basic Feasible Solution

$x_{11} = 5$, $x_{14} = 2$, $x_{23} = 7$, $x_{24} = 2$, $x_{32} = 8$, $x_{34} = 10$

The transportation cost is 5 (19) + 2 (10) + 7 (40) + 2 (60) + 8 (8) + 10 (20) = Rs. 779

2.

		Stores				Availability
		I	II	III	IV	
	A	21	16	15	13	11
Warehouse	B	17	18	14	23	13
	C	32	27	18	41	19
Requirement		6	10	12	15	

Solution

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	(13)	11	2
Warehouse	B	(17)	(18)	(14)	(23)	13	3
	C	(32)	(27)	(18)	(41)	19	9
Requirement		6	10	12	15		
Penalty		4	2	1	10		

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	11/0	2
Warehouse	B	(17)	(18)	(14)	(23)	13	3
	C	(32)	(27)	(18)	(41)	19	9
Requirement		6	10	12	15/4		
Penalty		4	2	1	10		

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	X	X
Warehouse	B	(17)	(18)	(14)	**4**(23)	13/9	3
	C	(32)	(27)	(18)	(41)	19	9
Requirement		6	10	12	15/4/0		
Penalty		15	9	4	18		

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	X	X
Warehouse	B	**6**(17)	(18)	(14)	**4**(23)	13/9/3	3
	C	(32)	(27)	(18)	(41)	19	9
Requirement		6/0	10	12	X		
Penalty		15	9	4	X		

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	X	X
Warehouse	B	**6**(17)	**3**(18)	(14)	**4**(23)	13/9/3/0	4
	C	(32)	(27)	(18)	(41)	19	9
Requirement		X	10/7	12	X		
Penalty		X	9	4	X		

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	X	X
Warehouse	B	**6**(17)	**3**(18)	(14)	**4**(23)	X	X
	C	(32)	**7**(27)	**12**(18)	(41)	X	X
Requirement		X	X	X	X		
Penalty		X	X	X	X		

Initial Basic Feasible Solution

$x_{14} = 11$, $x_{21} = 6$, $x_{22} = 3$, $x_{24} = 4$, $x_{32} = 7$, $x_{33} = 12$

The transportation cost is 11 (13) + 6 (17) + 3 (18) + 4 (23) + 7 (27) + 12 (18) = Rs. 796