FORTRAN 90

Lecturer : Rafel Hekmat Hameed
University of Babylon
Subject : Fortran 90
College of Engineering
Year : Second B.Sc.
Mechanical Engineering Dep.

Solution a set of linear equations by *Inverse Matrix*

One of the most important applications of matrices is to the solution of linear simultaneous equations.

Given

$$AX = B$$

We can multiply both sides by the inverse of A, provided this exists, to give

$$A^{-1}AX = A^{-1}B$$

But $A^{-1}A = I$, the identity matrix. Furthermore, $IX = X$, because multiplying any matrix by an identity matrix of the appropriate size leaves the matrix unaltered. So

$$X = A^{-1}B$$

This result gives us a method for solving simultaneous equations. All we need do is write them in matrix form, calculate the inverse of the matrix of coefficients, and finally perform a matrix multiplication.

Example

Solve the simultaneous equations

$X_1 + X_2 - 3X_3 = 9$

$-X_1 + 2X_2 = 6$

$X_1 - X_2 + X_3 = -5$

Provided you understand how matrices are multiplied together you will realize that these can be written in matrix form as

$$\begin{bmatrix} 1 & 1 & -3 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -5 \end{bmatrix}$$

We need to calculate the inverse of A

$$A^{-1}=\begin{bmatrix} 1/3 & 1/3 & 1 \\ 1/6 & 2/3 & 1/2 \\ -1/6 & 1/3 & 1/2 \end{bmatrix}$$

Then X is given by $X = A^{-1}B$

$$=\begin{bmatrix} 1/3 & 1/3 & 1 \\ 1/6 & 2/3 & 1/2 \\ -1/6 & 1/3 & 1/2 \end{bmatrix}\begin{bmatrix} 9 \\ 6 \\ -5 \end{bmatrix}$$

$$=\begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}$$

Hence $X_1 = 0$, $X_2 = 3$ and $X_3 = -2$ is the solution of the simultaneous equations.

A Fortran 90 program to solve a set of linear equations by using *INVERSE MATRIX*

```
program solve_equations_by_inverse_matrix
implicit none
integer,parameter::n=3
real,dimension (n)::b,x
real,dimension(n,n)::a,a1
integer::i,j,k,l
real::z
data a/1,-1,1,1,2,-1,-3,0,1/
data a1/1,3*0,1,3*0,1/
data b/9,6,-5/

!divided all elements of a & a1 by a(i,i)
do i=1,n
z=a(i,i)
do j=1,n
a(i,j)=a(i,j)/z
a1(i,j)=a1(i,j)/z
enddo

!make zero all entries in column a(j,i) & a1(j,i)
do j=i+1,n
z=a(j,i)
do k=1,n
a(j,k)=a(j,k)-z*a(i,k)
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a1(j,k)=a1(j,k)-z*a1(i,k)
enddo
enddo              ;          enddo

!subtract appropiate multiple of row j from j-1
do i=1,n-1
do j=i+1,n
z=a(i,j)
do l=1,n
a(i,l)=a(i,l)-z*a(j,l)
a1(i,l)=a1(i,l)-z*a1(j,l)
enddo
enddo
enddo

do i=1,n
write(*,60)(a(i,j),j=1,n) , (a1(i,j),j=1,n)
60 format(2x,3(f10.6),10x,3(f10.6))
enddo
print*,"********************"

CALL MULTI (a1,b,x)
do i=1,n
print 10,"x",i,"=",x(i)
enddo
10 format(2x,a,i1,a,f10.5)
end

subroutine multi(a1,b,x)
implicit none
integer,parameter::n=3
real,dimension (n)::b,x
real,dimension(n,n)::a1
integer::i,j,k
do i=1,n
do j=1,n
x(i)=0
do k=1,n
x(i)=x(i)+a1(i,k)*b(k)
enddo;enddo;enddo
end
```