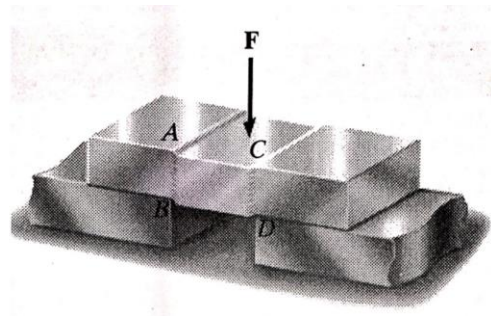
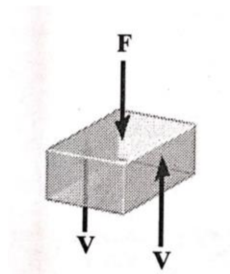
Average Shear stress

Shear stress is the stress component that acts in the plane of the sectional area. Consider a force F acting on the bar shown, if the Supports are rigid and the force is large enough, the material of the bar will deform and fail along the planes AB and CD.



By drawing a free-body-diagram for the unsupported center segment of the bar show that $V = {}^{F}/_{2}$



the Average shear stress distributed over each area is

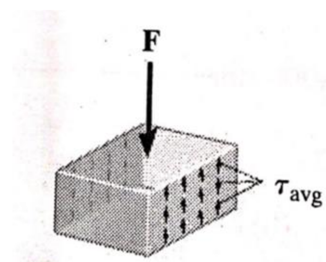
$$\boldsymbol{\tau_{avg} = \frac{V}{A}}$$

τ_{avg} is average shear stress at the section, assumed to be the same at each point located in the section

V is the internal resultant shear force on the section determined from the equation of equilibrium.

A is the area at the section.

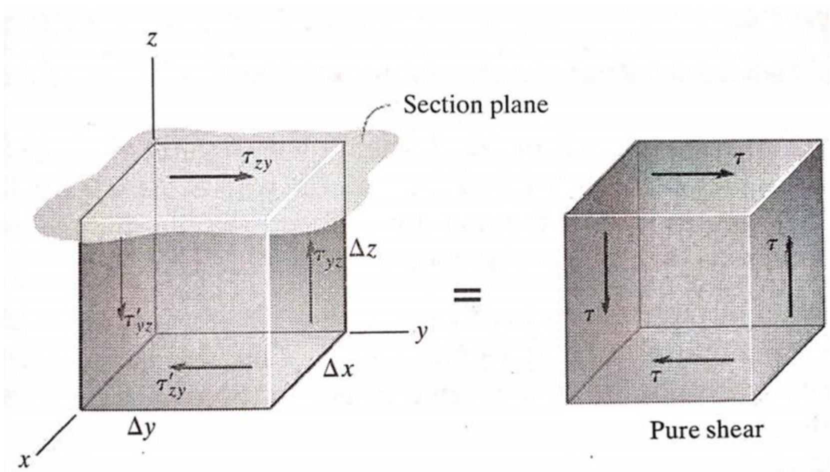
τ_{avg} is in the same direction of V .



(this loading case is an example of **Simple or Direct Shear stress**. This type occurs at simple connections that use bolts, pins, welding materials. However, this equation is approximate).

Shear stress Equilibrium

If we take a volume element at a point on the surface subjected to a shear stress τ_{zy}, the force and moment equilibrium requires τ_{zy} to accompanied by shear stresses acting on three other cases (τ'_{zy} , τ_{yz} , τ'_{yz}). Using equations of equilibrium we will show that these stresses are equal:



$\sum F_y = 0 \qquad \tau_{zy}(\Delta x \Delta y) - \tau'_{zy}(\Delta x \Delta y) = 0 \qquad \tau_{zy} = \tau'_{zy}$

$\sum F_z = 0 \qquad \tau_{yz}(\Delta x \Delta z) - \tau'_{yz}(\Delta x \Delta z) = 0 \qquad \tau_{yz} = \tau'_{yz}$

$\sum M_x = 0 \qquad -\tau_{zy}(\Delta_x \Delta_y)\Delta_z + \tau_{yz}(\Delta_x \Delta_z)\Delta_y = 0 \qquad \tau_{zy} = \tau_{yz}$

So that $\qquad \tau_{zy} = \tau'_{zy} = \tau_{yz} = \tau'_{yz} = \tau$

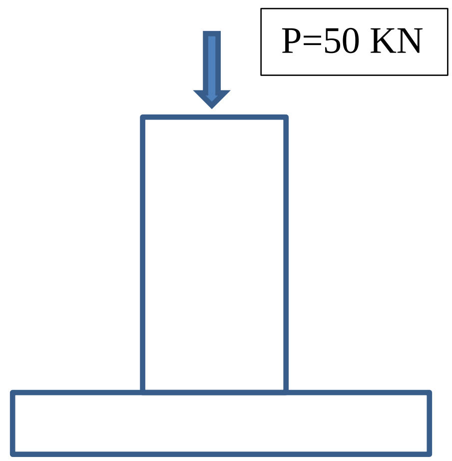
In other words, ***all four shear stresses must have equal magnitude and be directed either toward or away from each other at opposite edges of the element***.

The material in condition shown above is subjected to a ***pure shear.***

Problem 1

Compute the average shear stress developed in a plate (10 thickness) under the action of a piston (40 mm diameter) subjected to a force of 50 KN.

Solution



$$A = \pi \times D \times t$$

$$\tau = \frac{V}{A} = \frac{50000N}{\pi \times 40mm \times 10mm} = 39.7\, N/mm^2$$

If the maximum allowable average shear stress of the plate metal is 60 MPa, find the force required to punch the plate.

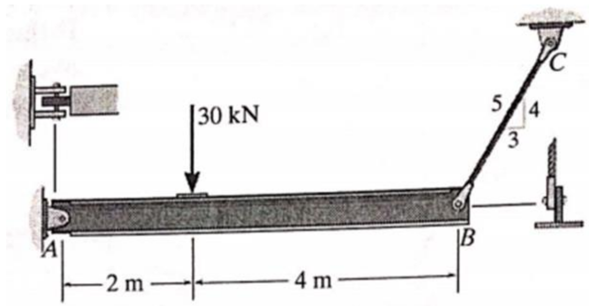
$$V = \tau \times A$$

$$V = 60\, {}^{N}/_{mm^2} \times (\pi \times 40mm \times 10mm) = 75360N$$

Problem 2

Determine the average shear stress in the 20-mm-diameter pin at A and the 30-mm-diameter pin at B that support the beam shown.

Solution

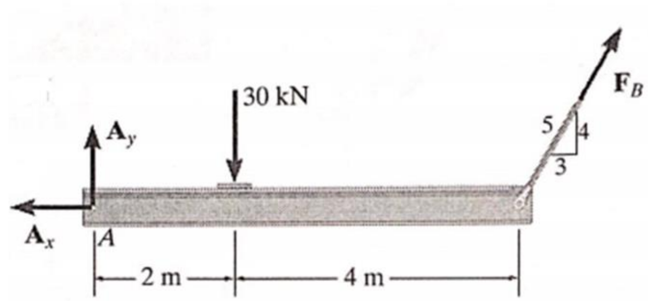


$\curvearrowleft + \sum M_A = 0$

$F_b\left(\frac{4}{5}\right)(6m) - 30kN(2m) = 0, F_b = 12.5kN$

$\overset{+}{\rightarrow} \sum F_x = 0$

$(12.5kN)\left(\frac{3}{5}\right) - A_x = 0,\ \ A_x = 7.5kN$

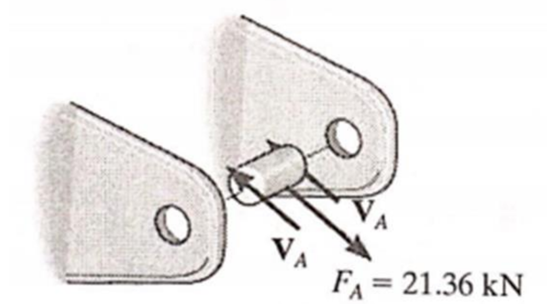


$+\uparrow \sum F_y = 0$

$A_y + (12.5kN)\left(\frac{4}{5}\right) - 30kN = 0, \qquad A_y = 20kN$

The resultant force acting on pin A

$F_a = \sqrt{A_x^2 + A_y^2} = \sqrt{(7.5kN)^2 + (20kN)^2} = 21.36kN$

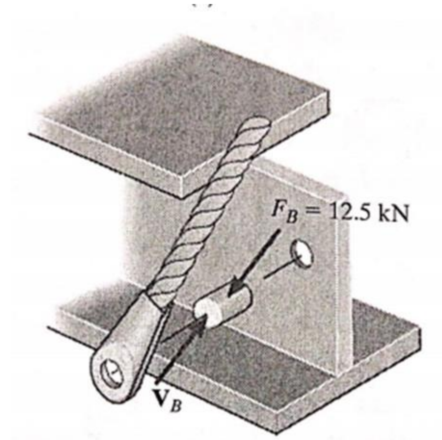


The pin at A is supported by two fixed leaves so:

$V_a = \frac{F_a}{2} = \frac{21.36kN}{2} = 10.68kN$

Pin B is subjected to a single shear

$V_b = F_b = 12.5kN$



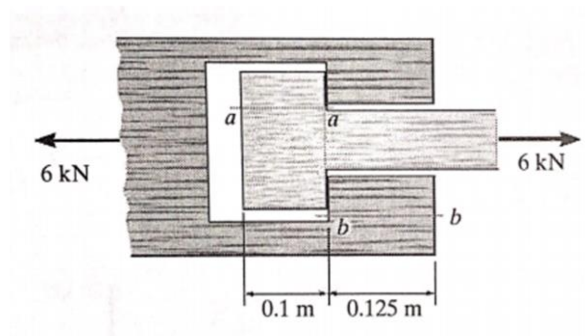
Average Shear stress

$$\tau_{a,avg} = \frac{V_a}{A_a} = \frac{10.68(10^3)N}{\frac{\pi}{4}(0.02m)^2} = 34.0MPa$$

$$\tau_{b,avg} = \frac{V_b}{A_b} = \frac{12.5(10^3)N}{\frac{\pi}{4}(0.03m)^2} = 17.7MPa$$

Problem 3

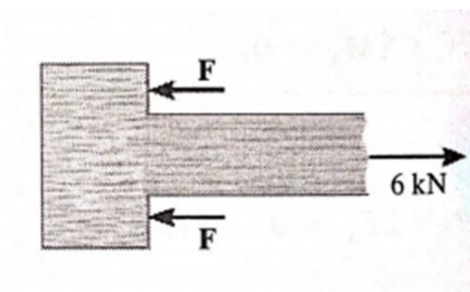
Suppose the wood joint below has a width of 150mm, determine the average shear stress developed along shear planes a-a and b-b. For each shear plane, represent the state of stress on an element of the material.



Solution

Free body diagram

$\overset{+}{\rightarrow} \sum F_x = 0\ ,\ \ 6kN - F - F = 0\ ,\ \ F = 3kN$



Consider equilibrium of segments cut at a-a and b-b

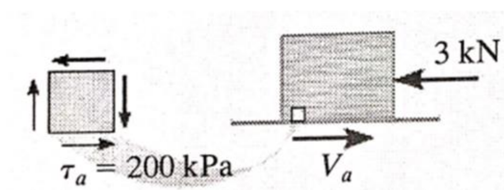
$\overset{+}{\rightarrow} \sum F_x = 0\ ,\ \ V_a - 3kN = 0\ ,\ \ V_a = 3kN$

$\overset{+}{\rightarrow} \sum F_x = 0\ ,\ \ 3kN - V_b = 0\ ,\ \ V_b = 3kN$

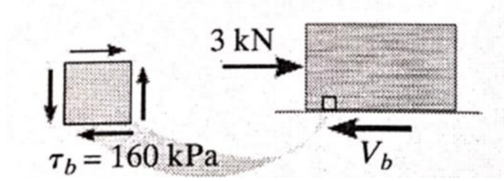
Average Shear stress

$$\tau_{a,avg} = \frac{V_a}{A_a} = \frac{3(10^3)N}{(0.1m)(0.15m)} = 200kPa$$

$$\tau_{b,avg} = \frac{V_b}{A_b} = \frac{3(10^3)N}{(0.125m)(0.15m)} = 160kPa$$

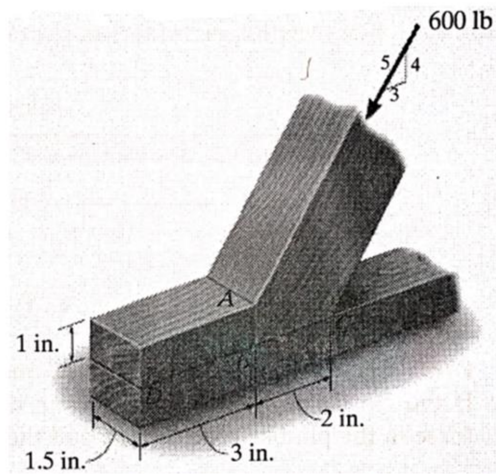


Representation of state stress, tao has to be at the direction of V and opposite to each other. This is a **pure shear stress**



Problem 4

The inclined member shown is subjected to a compressive force of 600lb. determine the average compressive stress along the smooth areas of contact defined by AB and BC, and the average shear stress along the horizontal plane defined by DB.

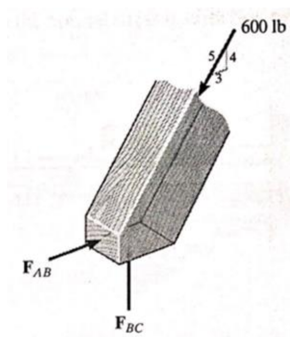


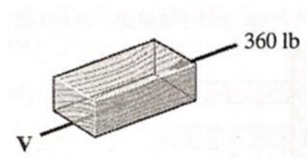
Solution

Free-body-diagram

$$\overset{+}{\rightarrow}\sum F_x = 0\ ,\ \ F_{ab} - 600lb\left(\frac{3}{5}\right) = 0\ ,\ \ F_{ab} = 360lb$$

$$\overset{+}{\rightarrow}\sum F_y = 0\ ,\ \ F_{cb} - 600lb\left(\frac{4}{5}\right) = 0\ ,\ \ F_{cb} = 480lb$$





Shear force acting on plane DB is

$$\overset{+}{\rightarrow}\sum F_x = 0\ ,\ V = 360lb$$

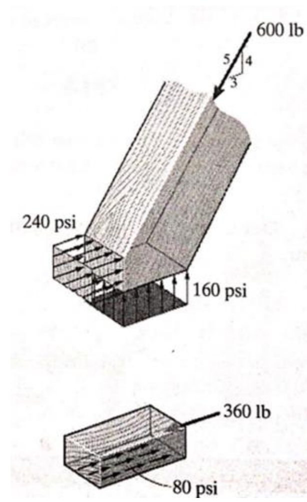
Average compressive normal stress

$$\sigma_{ab} = \frac{F_{ab}}{A_{ab}} = \frac{360lb}{(1in.)(1.5in.)} = 240psi$$

$$\sigma_{bc} = \frac{F_{bc}}{A_{bc}} = \frac{480lb}{(2in.)(1.5in.)} = 160psi$$

Average shear stress

$$\tau_{avg} = \frac{360ib}{(3in.)(1.5in.)} = 80psi$$



Bearing Stress

Bearing stress, as previously defined, is the contact pressure between two separate bodies (different from compressive normal stress, as it is an internal stress caused by compressive force).

Occurs at (bearing plat, bolted or riveted connections, ..)

$$\boldsymbol{\sigma_b = \frac{P_b}{A_b}}$$

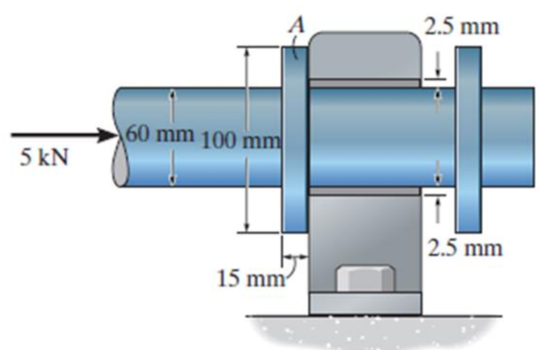
σ_b is the bearing stress at the contact area

P_b is the contact force

A_b is the contact area

Problem 5

if the shaft shown below is subjected to an axial force of 5kN, determine the bearing stress acting on the collar A.



Solution

$\overset{+}{\rightarrow} \sum F_x = 0, \quad P_b = 5kN$

$A_b = \pi(0.5^2 - 0.0325^2) = 4.536(10^{-3})m^2$

$\sigma_b = \frac{P_b}{A_b} = \frac{5000N}{4.536(10^{-3})m^2} = 1.1MPa$

Allowable Stress

It is necessary to restrict the stress in the material to a level that will be safe. (for uncertainty of loading, material, weathering, and mistakes).

One method to specify the allowable load is factor of Safety (F.S):

$$F.S = \frac{F_{fail}}{F_{allow}}$$

For the simple stresses we have been studying, the load is linearly related to the stress developed in the member, we can also express factor of safety as a ratio of the failure stress to the allowable stress.

$$F.S = \frac{\sigma_{fail}}{\sigma_{allow}} \; , \qquad\qquad F.S = \frac{\tau_{fail}}{\tau_{allow}}$$

Design of simple connections

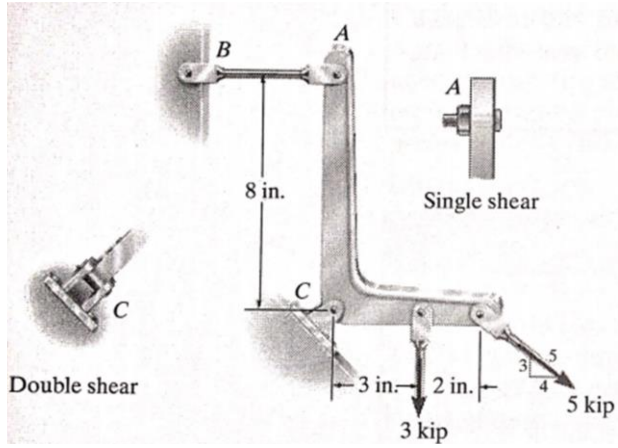
Expressions of average normal stress and average shear stress can be used to calculate the required area and by the way the section dimensions that withstand the loads.

$$A = \frac{P}{\sigma_{allow}} \; , \qquad or \; , \qquad A = \frac{V}{\tau_{allow}}$$

Problem 6

The control arm is subjected to the loading shown; determine, to the nearest one-quarter on an inch, the required diameter of the steel pin at C if the allowable shear stress for the steel is $\tau_{allow} = 8ksi$.

Solution



Free-body-diagram

$\curvearrowleft + \sum M_A = 0$

$F_{ab}(8in.) - 3kip(3in.) - 5kip\left(\frac{3}{5}\right)(5in.) = 0$

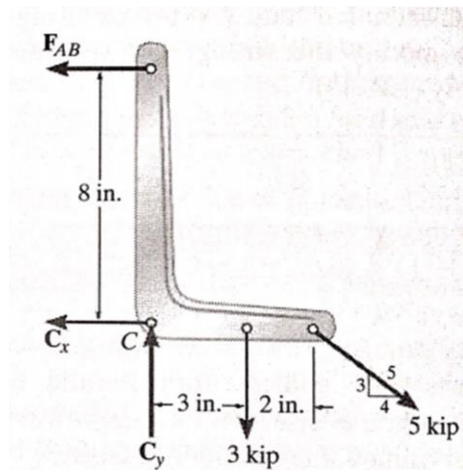
$F_{ab} = 3kip$

$\overset{+}{\rightarrow} \sum F_x = 0$

$-3kip - C_x + 5kip\left(\frac{4}{5}\right) = 0, \quad C_x = 1kip$

$+\uparrow \sum F_y = 0$

$C_y - 3kip - 5kip\left(\frac{3}{5}\right) = 0, \quad C_y = 6kip$



The resultant of forces at C

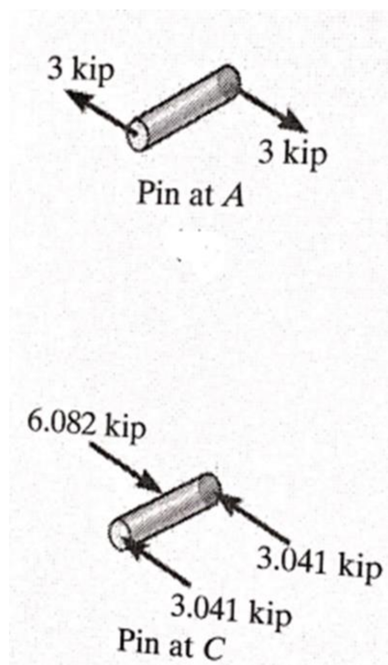
$F_c = \sqrt{(1kip)^2 + (6kip)^2} = 6.082kip$

The pin is subjected to double shear, so V=3.041kip.

$A = \frac{V}{\tau_{allow}} = \frac{3.041kip}{8\,kip/in.^2} = 0.3802in.^2$

$\pi\left(\frac{d}{2}\right)^2 = 0.3082in.^2, \quad d = 0.696in.$

Practically steel pins is available to the nearest quarter of an inch, so use d=0.75 in.



Homework

On your text solve problems **(1.31, 1.44, 1.48, 1.51, 1.73, and 1.81)**