5.5 STEP RESPONSE OF AN *RC* CIRCUIT

When the dc source of an ***RC*** circuit is suddenly applied, the voltage or current source can be modeled as a step function, and the response is known as a ***step response***.

The step response of a circuit is its behavior when the excitation is the step function, which may be a voltage or a current source.

The step response is the response of the circuit due to a sudden application of a dc voltage or current source.

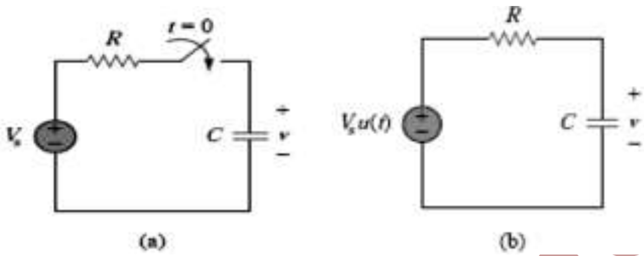


Figure 5.10 An *RC* circuit with voltage step input.

Consider the ***RC*** circuit in **Fig. 5.10(a)** which can be replaced by the circuit in **Fig. 5.10(b)**, where ***Vs*** is a constant, dc voltage source. Again, we select the capacitor voltage as the circuit response to be determined.

We assume an initial voltage V_0 on the capacitor, although this is not necessary for the step response. Since the voltage of a capacitor cannot change instantaneously,

$$v(0^-) = v(0^+) = V0 \qquad (7.22)$$

where $v(0-)$ is the voltage across the capacitor just before switching and $v(0+)$ is its voltage immediately after switching. Applying **KCL**, we have

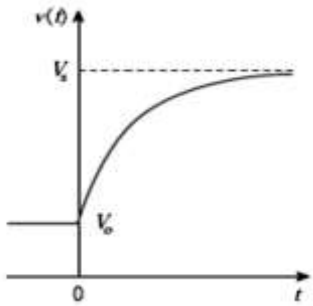
$$C\frac{dv}{dt} + \frac{v - Vs.u(t)}{R} = 0 \qquad (7.23)$$

where ***v*** is the voltage across the capacitor.

Thus,

$$v(t) = \begin{cases} V0 & ,t < 0 \\ Vs + (V0 - Vs)e^{-t/\tau} & ,t > 0 \end{cases} \qquad (5.24)$$

This is known as the *complete response* of the *RC* circuit to a sudden application of a dc voltage source, assuming the capacitor is initially charged. The reason for the term “complete” will become evident a little later. Assuming that $Vs > V0$, a plot of ***v(t)*** is shown in **Fig. 5.11**.



If we assume that the capacitor is uncharged initially, we set $V0 = 0$ in **Eq. (5.24)** so that

Figure 5.11 Response of an *RC* circuit with initially charged capacitor.

$$v(t) = \begin{cases} 0 & ,t < 0 \\ Vs\,(1 - e^{-t/\tau}), & t > 0 \end{cases} \tag{5.25}$$

Rather than going through the derivations above, there is a systematic approach—or rather, a short-cut method—for finding the step response of an ***RC*** or ***RL*** circuit. Let us reexamine **Eq. (5.24)**, which is more general than **Eq. (5.25)**. It is evident that ***v(t)*** has two components. Thus, we may write

$$v = v_f + v_n \tag{5.26}$$

We know that ***vn*** is the natural response of the circuit, as discussed in **Section 5.2**. Now, ***vf*** is known as the *forced* response because it is produced by the circuit when an external "force" is applied (a voltage source in this case).

The natural response or transient response is the circuit's temporary response that will die out with time.

The forced response or steady-state response is the behavior of the circuit a long time after an external excitation is applied.

The complete response of the circuit is the sum of the natural response and the forced response. Therefore, we may write **Eq. (5.24)** as

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \tag{5.27}$$

where ***v*(0)** is the initial voltage at $t = 0^+$ and ***v(∞)*** is the final or steady state value. Thus, to find the step response of an ***RC*** circuit requires three things:

1. The initial capacitor voltage ***v* (0)**.
2. The final capacitor voltage ***v* (∞)**.
3. The time constant ***τ***.

Note that if the switch changes position at time $t = t0$ instead of at $t = 0$, there is a time delay in the response so that **Eq. (5.27)** becomes

$$v(t) = v(\infty) + [v(t0) - v(\infty)]e^{-(t-t0)/\tau} \tag{5.28}$$

where ***v*(*t*0)** is the initial value at $t = t^{+0}$. Keep in mind that **Eq. (5.27)** or **(5.28)** applies only to step responses, that is, when the input excitation is constant.

Example 5.10: The switch in **Fig. 5.12** has been in position *A* for a long time. At $t = 0$, the switch moves to *B*. Determine $v(t)$ for $t > 0$ and calculate its value at $t = 1$ s and 4 s.

Solution:

For $t < 0$, the switch is at position *A*. Since v is the same as the voltage across the 5-kΩresistor, the voltage across the capacitor just before $t = 0$ is obtained by voltage division as

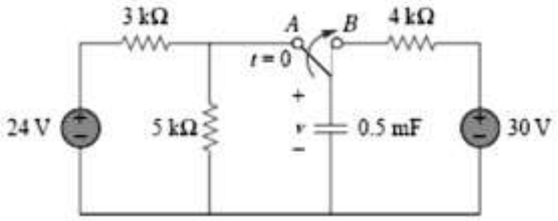


Figure 5.12 For Example 5.10.

$$v(0^-) = \frac{5}{5+3}(24) = 15\,V$$

Using the fact that the capacitor voltage cannot change instantaneously,

$$v(0) = v(0^-) = v(0^+) = 15 \text{ V}$$

For $t > 0$, the switch is in position *B*. The Thevenin resistance connected to the capacitor is $R_{Th} = 4$ kΩ, and the time constant is

$$\tau = R_{Th}C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2 \text{ s}$$

Since the capacitor acts like an open circuit to dc at steady state, $v(\infty) = 30$ V. Thus,

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = 30 + (15 - 30)e^{-t/2} = (30 - 15e^{-0.5t}) \text{ V}$$

At $t = 1$, $v(1) = 30 - 15e^{-0.5} = 20.902$ V

At $t = 4$, $v(4) = 30 - 15e^{-2} = 27.97$ V

Notice that the capacitor voltage is continuous while the resistor current is not.

PRACTICE PROBLEM:

(1) Find $v(t)$ fort > 0 in the circuit in Figure below. Assume the switch has been open for a long time and is closed at $t = 0$. Calculate $v(t)$ at $t = 0.5$.

Answer: $-5 + 15e^{-2t}$ V, 0.5182 V.