

A **digital system** can understand positional number system only where there are a few symbols called digits and these symbols represent different values depending on the position they occupy in the number.

A value of each digit in a number can be determined using

- The digit
- The position of the digit in the number
- The base of the number system (where base is defined as the total number of digits available in the number system).

Decimal Number System

- The number system that we use in our day-to-day life is the decimal number system. Decimal number system has base 10 as it uses 10 digits from 0 to 9. In decimal number system, the successive positions to the left of the decimal point represents units, tens, hundreds, thousands and so on.
- Each position represents a specific power of the base (10). For example, the decimal number 1234 consists of the digit 4 in the units position, 3 in the tens position, 2 in the hundreds position, and 1 in the thousands position, and its value can be written as
 - $(1 \times 1000) + (2 \times 100) + (3 \times 10) + (4 \times 1)$
 - $(1 \times 10^3) + (2 \times 10^2) + (3 \times 10^1) + (4 \times 10^0)$
 - $1000 + 200 + 30 + 1$
 - 1234

The following number systems are frequently used in computers

1 **Binary Number System:** Base 2. Digits used: 0, 1

2 **Octal Number System:** Base 8. Digits used: 0 to 7

3 **Hexa Decimal Number System:** Base 16. Digits used: 0 to 9, Letters used: A- F

Binary Number System

- Characteristics
- Uses two digits, 0 and 1.
- Also called base 2 number system
- Each position in a binary number represents a 0 power of the base (2). Example: 2^0
- Last position in a binary number represents an x power of the base (2). Example: 2^x where x represents the last position - 1.

Example

Binary Number: 10101_2
Calculating Decimal Equivalent

| Step | Binary Number | Decimal Number |
|--------|---------------|---|
| Step 1 | 10101_2 | $((1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$ |
| Step 2 | 10101_2 | $(16 + 0 + 4 + 0 + 1)_{10}$ |
| Step 3 | 10101_2 | 21_{10} |


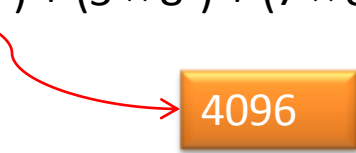
Note: 10101_2 is normally written as 10101

Octal Number System

- Characteristics
- Uses eight digits, 0,1,2,3,4,5,6,7.
- Also called base 8 number system
- Each position in an octal number represents a 0 power of the base (8). Example: 8^0
- Last position in an octal number represents an x power of the base (8). Example: 8^x where x represents the last position - 1.

Example

Octal Number – 12570_8
Calculating Decimal Equivalent

| Step | Octal Number | Decimal Number |
|--------|--------------|--|
| Step 1 | 12570_8 | $((1 \times 8^4) + (2 \times 8^3) + (5 \times 8^2) + (7 \times 8^1) + (0 \times 8^0))_{10}$  |
| Step 2 | 12570_8 | $(4096 + 1024 + 320 + 56 + 0)_{10}$  |
| Step 3 | 12570_8 | 5496_{10} |

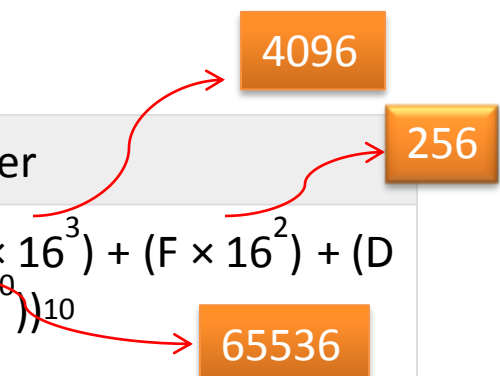
Note: 12570_8 is normally written as 12570.

Hexadecimal Number System

- Characteristics
- Uses 10 digits and 6 letters, 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Letters represents numbers starting from 10. A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.
- Also called base 16 number system.
- Each position in a hexadecimal number represents a 0 power of the base (16). Example 16^0 .
- Last position in a hexadecimal number represents an x power of the base (16). Example 16^x where x represents the last position - 1.

Example

Hexadecimal Number: $19FDE_{16}$
Calculating Decimal Equivalent

| Step | Binary Number | Decimal Number |
|--------|---------------|---|
| Step 1 | $19FDE_{16}$ | $((1 \times 16^4) + (9 \times 16^3) + (F \times 16^2) + (D \times 16^1) + (E \times 16^0))_{10}$  |
| Step 2 | $19FDE_{16}$ | $((1 \times 16^4) + (9 \times 16^3) + (15 \times 16^2) + (13 \times 16^1) + (14 \times 16^0))_{10}$ |
| Step 3 | $19FDE_{16}$ | $(65536 + 36864 + 3840 + 208 + 14)_{10}$ |
| Step 4 | $19FDE_{16}$ | 106462_{10} |

Note – $19FDE_{16}$ is normally written as 19FDE.

Decimal to Other Base System

- Steps
- **Step 1** – Divide the decimal number to be converted by the value of the new base.
- **Step 2** – Get the remainder from Step 1 as the rightmost digit (least significant digit) of new base number.
- **Step 3** – Divide the quotient of the previous divide by the new base.
- **Step 4** – Record the remainder from Step 3 as the next digit (to the left) of the new base number.
- Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.
- The last remainder thus obtained will be the Most Significant Digit (MSD) of the new base number.

EXAMPLE 1

Decimal Number: 29_{10}
Calculating Binary Equivalent

| Step | Operation | Result | Remainder |
|--------|-----------|--------|-----------|
| Step 1 | $29 / 2$ | 14 | 1 |
| Step 2 | $14 / 2$ | 7 | 0 |
| Step 3 | $7 / 2$ | 3 | 1 |
| Step 4 | $3 / 2$ | 1 | 1 |
| Step 5 | $1 / 2$ | 0 | 1 |

$$((1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10} = 29$$

As mentioned in Steps 2 and 4, the remainders have to be arranged in the reverse order so that the first remainder becomes the Least Significant Digit (LSD) and the last remainder becomes the Most Significant Digit (MSD).

Decimal Number – 29_{10} = Binary Number – 11101_2 .

EXAMPLE 2

Decimal Number: 15_{10}
Calculating Binary Equivalent

| Step | Operation | Result | Remainder |
|--------|-----------|--------|---|
| Step 1 | $15 / 2$ | 7 | 1 |
| Step 2 | $7 / 2$ | 3 | 1 |
| Step 3 | $3 / 2$ | 1 | 1 |
| Step 4 | $1 / 2$ | 0 | 1 |
| | | | $((1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0))_{10} = 15$ |

As mentioned in Steps 2 and 4, the remainders have to be arranged in the reverse order so that the first remainder becomes the Least Significant Digit (LSD) and the last remainder becomes the Most Significant Digit (MSD).

Decimal Number – 29_{10} = Binary Number – 1111_2 .

Other Base System to Decimal System

- Steps
- **Step 1** – Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system).
- **Step 2** – Multiply the obtained column values (in Step 1) by the digits in the corresponding columns.
- **Step 3** – Sum the products calculated in Step 2. The total is the equivalent value in decimal.

Example

Binary Number – 11101_2
Calculating Decimal Equivalent

| Step | Binary Number | Decimal Number |
|--|---------------|---|
| Step 1 | 11101_2 | $((1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$ |
| Step 2 | 11101_2 | $(16 + 8 + 4 + 0 + 1)_{10}$ |
| Step 3 | 11101_2 | 29_{10} |
| Binary Number – $11101_2 =$ Decimal Number – 29_{10} | | |