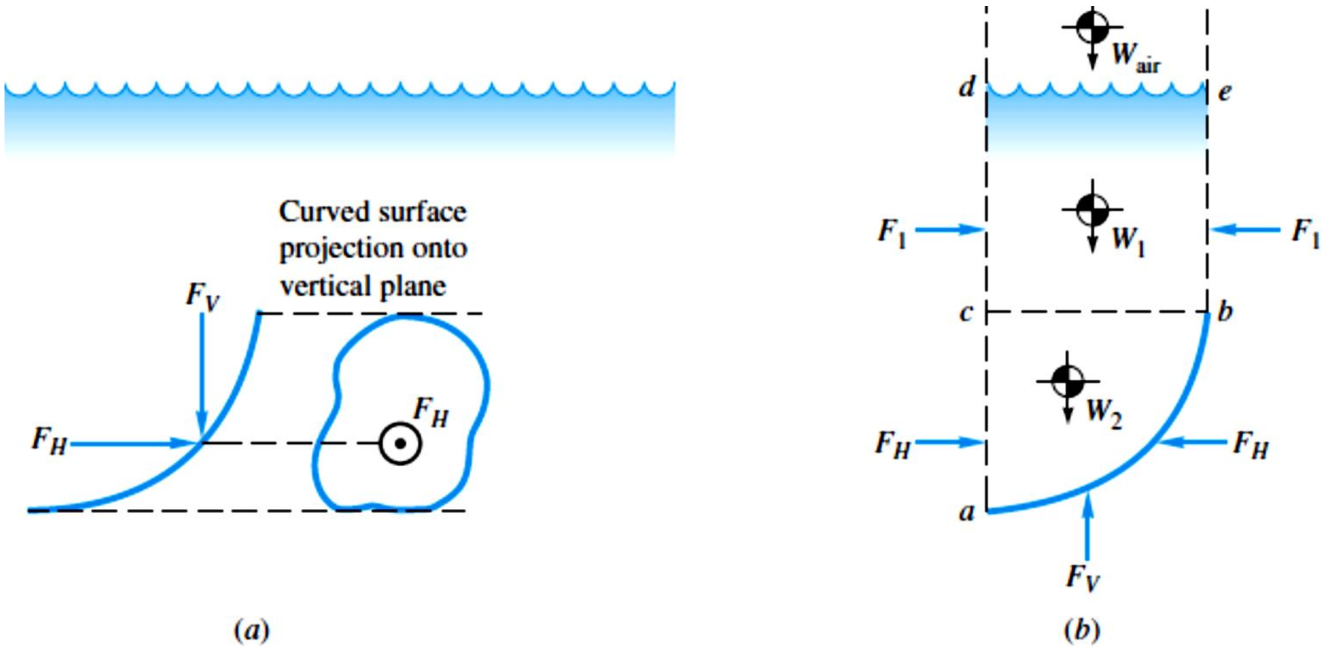
7.th week

Hydrostatic forces on curved surfaces



The resultant pressure force on a curved surface is computed by separating it into horizontal and vertical components, as shown in the **Figure a**.

Fig. b shows a free-body diagram of the column of fluid contained in the vertical projection above the curved surface.

Forces on the vertical projection above the curved surface are:

1- F_H & F_V= pressure force components on the curved portion.

2- F_1= pressure force on the vertical surface.

3- W_1 & W_2 = fluid weight over the curved surface

On the upper part ***bcde***, the force (***F_1***) is balanced by (F_1) on the opposite side.

On the lower part, the horizontal force is computed by:

$FH_{Pressure}$= horizontal component of ($P_{CG}A$)

The horizontal component of force on a curved surface equals the force on the plane area formed by the projection of the curved surface onto a vertical plane normal to the component.

If there are two or more horizontal forces they will be calculated by the same way.

Summation of vertical forces on the fluid free body then shows that

$$F_V = W_1 + W_2 + W_{air} \tag{2.45}$$

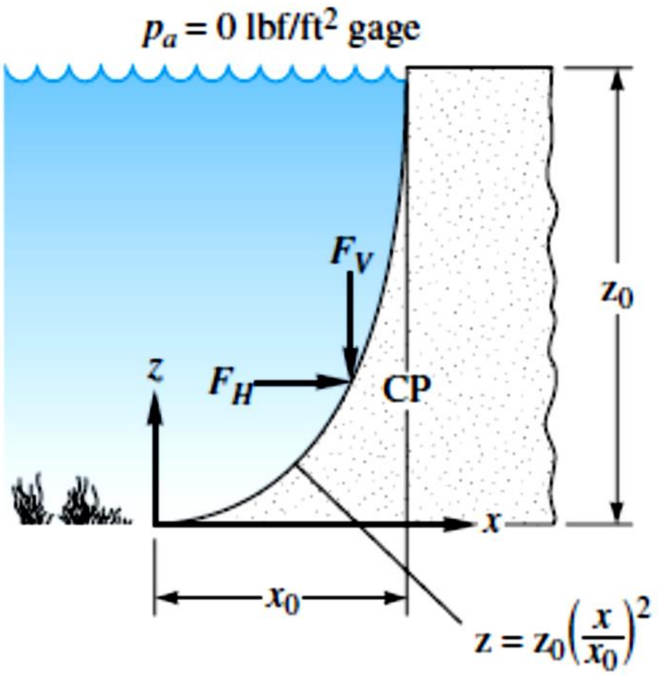
We can state this in words as our second general rule:

The vertical component of pressure force on a curved surface equals in magnitude and direction the weight of the entire column of fluid, both liquid and atmosphere, above the curved surface.

The calculation of F_V involves finding centers of mass of the fluid column. This need integration of area of the lower part *abc* when it has irregular shape.

EXAMPLE 2.7

A dam has a parabolic shape $z/z_0 = (x/x_0)^2$ as shown in Fig. E2.7*a*, with $x_0 = 10$ ft and $z_0 = 24$ ft. The fluid is water, $\gamma = 62.4$ lbf/ft^3, and atmospheric pressure may be omitted. Compute the forces F_H and F_V on the dam and the position CP where they act. The width of the dam is 50 ft.



E2.7a

Solution

The vertical projection of this curved surface is a rectangle 24 ft high and 50 ft wide, with its centroid halfway down, or $h_{CG} = 12$ ft. The force F_H is thus

$$F_H = \gamma h_{CG} A_{proj} = (62.4 \text{ lbf/ft}^3)(12 \text{ ft})(24 \text{ ft})(50 \text{ ft})$$

$$= 899{,}000 \text{ lbf} = 899 \times 10^3 \text{ lbf} \qquad \textit{Ans.}$$

The line of action of F_H is below the centroid by an amount

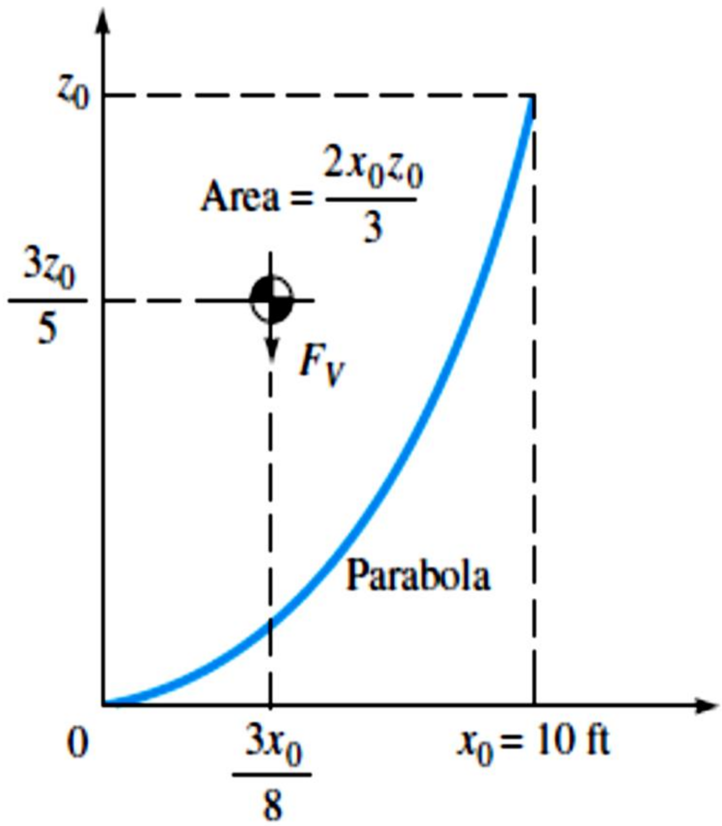
$$y_{CP} = -\frac{I_{xx} \sin\theta}{h_{CG} A_{proj}} = -\frac{\frac{1}{12}(50 \text{ ft})(24 \text{ ft})^3(\sin 90°)}{(12 \text{ ft})(24 \text{ ft})(50 \text{ ft})} = -4 \text{ ft}$$

Thus F_H is $12 + 4 = 16$ ft, or two-thirds, down from the free surface or 8 ft from the bottom, as might have been evident by inspection of the triangular pressure distribution.

The vertical component F_V equals the weight of the parabolic portion of fluid above the curved surface. The geometric properties of a parabola are shown in Fig. E2.7*b*. The weight of this amount of water is

$$F_V = \gamma(\tfrac{2}{3}x_0 z_0 b) = (62.4 \text{ lbf/ft}^3)(\tfrac{2}{3})(10 \text{ ft})(24 \text{ ft})(50 \text{ ft})$$

$$= 499{,}000 \text{ lbf} = 499 \times 10^3 \text{ lbf} \qquad \textit{Ans.}$$



E2.7b

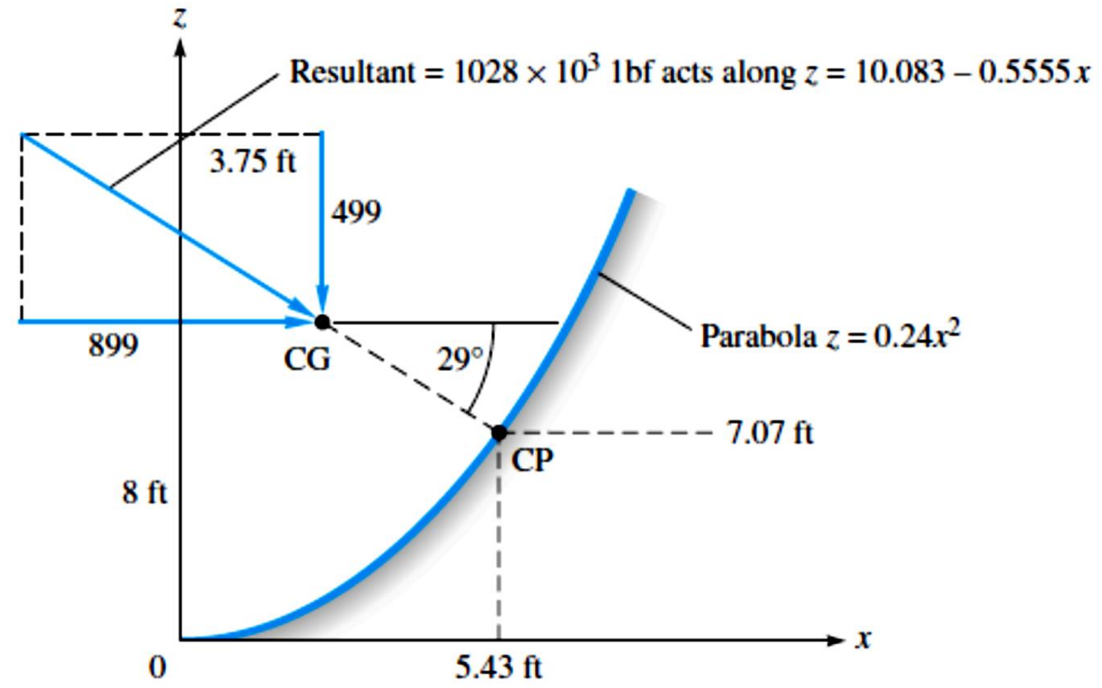
This acts downward on the surface at a distance $3x_0/8 = 3.75$ ft over from the origin of coordinates. Note that the vertical distance $3z_0/5$ in Fig. E2.7*b* is irrelevant.

The total resultant force acting on the dam is

$$F = (F_H^2 + F_V^2)^{1/2} = [(499)^2 + (899)^2]^{1/2} = 1028 \times 10^3 \text{ lbf}$$

As seen in Fig. E2.7*c*, this force acts down and to the right at an angle of $29° = \tan^{-1} \frac{499}{899}$. The force F passes through the point $(x, z) = (3.75 \text{ ft}, 8 \text{ ft})$. If we move down along the 29° line until we strike the dam, we find an equivalent center of pressure on the dam at

$$x_{CP} = 5.43 \text{ ft} \qquad z_{CP} = 7.07 \text{ ft} \qquad \textit{Ans.}$$



E2.7c

Hydrostatic forces in layered fluids

The formulas for plane and curved surfaces in Secs. 2.5 and 2.6 are valid only for a fluid of uniform density. If the fluid is layered with different densities, as in Fig. 2.15, a single formula cannot solve the problem because the slope of the linear pressure distribution changes between layers. However, the formulas apply separately to each layer, and thus the appropriate remedy is to compute and sum the separate layer forces and moments.

Consider the slanted plane surface immersed in a two-layer fluid in Fig. 2.15. The slope of the pressure distribution becomes steeper as we move down into the denser second layer. The total force on the plate does *not* equal the pressure at the centroid times the plate area, but the plate portion in each layer does satisfy the formula, so that we can sum forces to find the total:

$$F = \sum F_i = \sum p_{CG_i} A_i \tag{2.46}$$

Similarly, the centroid of the plate portion in each layer can be used to locate the center of pressure on that portion

$$y_{CP_i} = -\frac{\rho_i g \sin\theta_i I_{xx_i}}{p_{CG_i} A_i} \qquad x_{CP_i} = -\frac{\rho_i g \sin\theta_i I_{xy_i}}{p_{CG_i} A_i} \tag{2.47}$$

These formulas locate the center of pressure of that particular F_i with respect to the centroid of that particular portion of plate in the layer, not with respect to the centroid of the entire plate. The center of pressure of the total force $F = \sum F_i$ can then be found by summing moments about some convenient point such as the surface. The following example will illustrate.

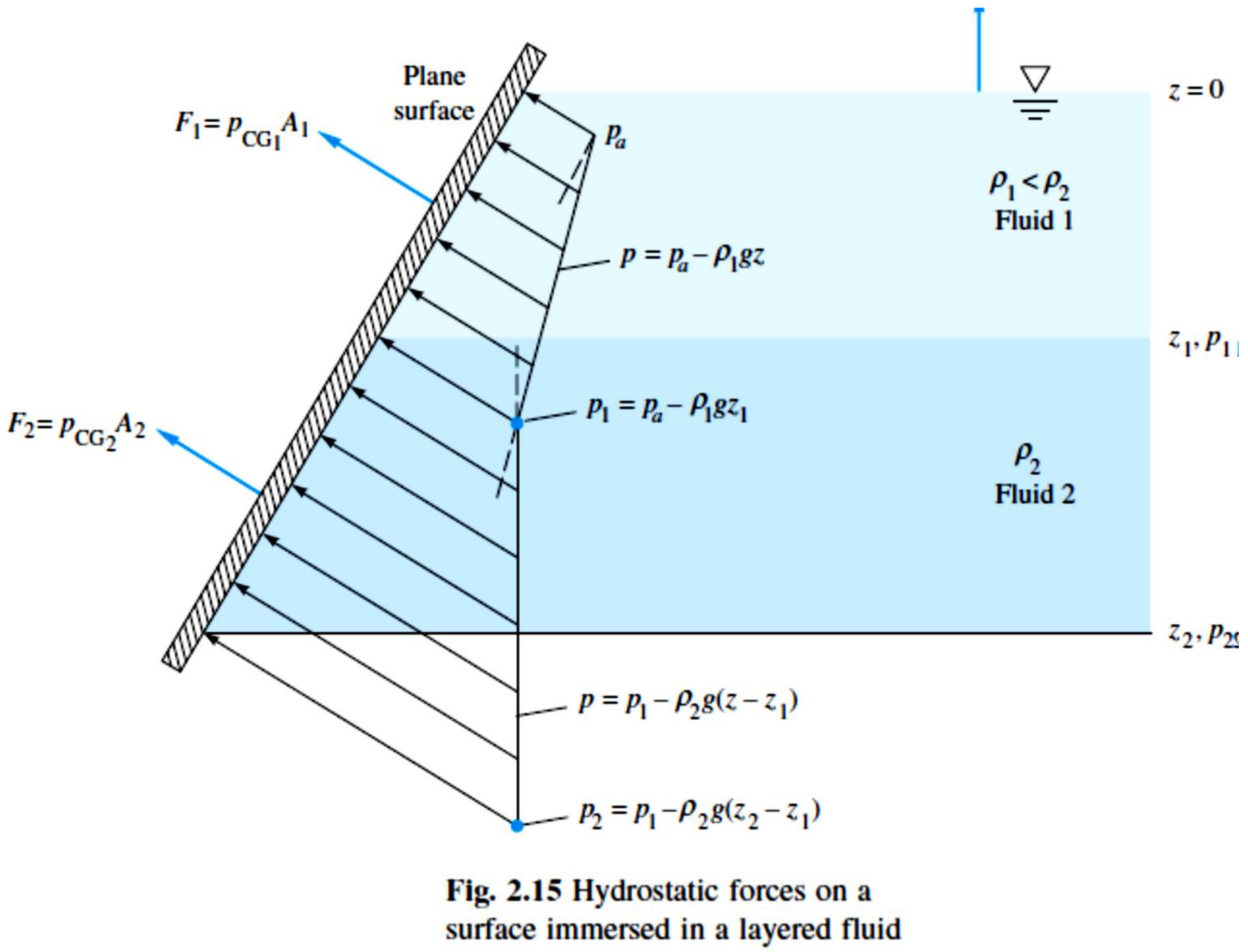


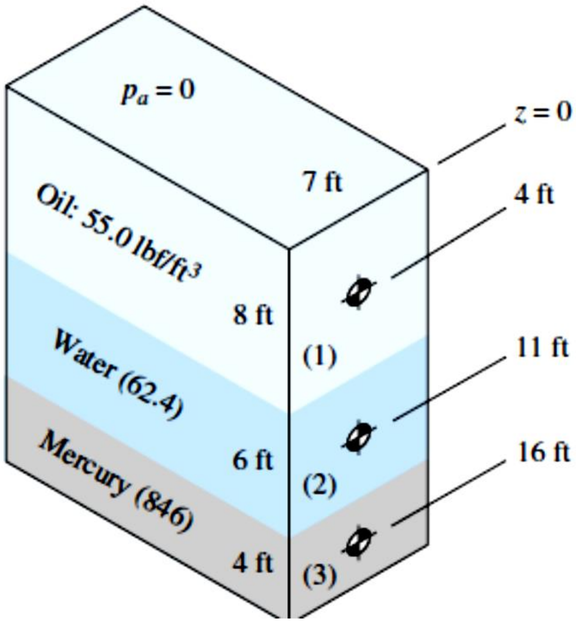
Fig. 2.15 Hydrostatic forces on a surface immersed in a layered fluid must be summed in separate pieces.

EXAMPLE 2.8

A tank 20 ft deep and 7 ft wide is layered with 8 ft of oil, 6 ft of water, and 4 ft of mercury. Compute (*a*) the total hydrostatic force and (*b*) the resultant center of pressure of the fluid on the right-hand side of the tank.

Solution

Divide the end panel into three parts as sketched in Fig. E2.8, and find the hydrostatic pressure at the centroid of each part, using the relation (2.38) in steps as in Fig. E2.8:



$$p_{CG_1} = (55.0\ \text{lbf/ft}^3)(4\ \text{ft}) = 220\ \text{lbf/ft}^2$$

$$p_{CG_2} = (55.0)(8) + 62.4(3) = 627\ \text{lbf/ft}^2$$

$$p_{CG_3} = (55.0)(8) + 62.4(6) + 846(2) = 2506\ \text{lbf/ft}^2$$

These pressures are then multiplied by the respective panel areas to find the force on each portion:

$$F_1 = p_{CG_1}A_1 = (220\ \text{lbf/ft}^2)(8\ \text{ft})(7\ \text{ft}) = 12{,}300\ \text{lbf}$$

$$F_2 = p_{CG_2}A_2 = 627(6)(7) = 26{,}300\ \text{lbf}$$

$$F_3 = p_{CG_3}A_3 = 2506(4)(7) = \underline{70{,}200\ \text{lbf}}$$

$$F = \Sigma F_i = 108{,}800\ \text{lbf} \qquad \textit{Ans. (a)}$$

Part b:

Equations (2.47) can be used to locate the CP of each force F_i, noting that $\theta = 90°$ and $\sin\theta = 1$ for all parts. The moments of inertia are $I_{xx_1} = (7\text{ ft})(8\text{ ft})^3/12 = 298.7\text{ ft}^4$, $I_{xx_2} = 7(6)^3/12 = 126.0\text{ ft}^4$, and $I_{xx_3} = 7(4)^3/12 = 37.3\text{ ft}^4$. The centers of pressure are thus at

$$y_{CP_1} = -\frac{\rho_1 g I_{xx_1}}{F_1} = -\frac{(55.0\text{ lbf/ft}^3)(298.7\text{ ft}^4)}{12{,}300\text{ lbf}} = -1.33\text{ ft}$$

$$y_{CP_2} = -\frac{62.4(126.0)}{26{,}300} = -0.30\text{ ft} \qquad y_{CP_3} = -\frac{846(37.3)}{70{,}200} = -0.45\text{ ft}$$

This locates $z_{CP_1} = -4 - 1.33 = -5.33$ ft, $z_{CP_2} = -11 - 0.30 = -11.30$ ft, and $z_{CP_3} = -16 - 0.45 = -16.45$ ft. Summing moments about the surface then gives

$$\sum F_i z_{CP_i} = F z_{CP}$$

or $$12{,}300(-5.33) + 26{,}300(-11.30) + 70{,}200(-16.45) = 108{,}800 z_{CP}$$

or $$z_{CP} = -\frac{1{,}518{,}000}{108{,}800} = -13.95\text{ ft} \qquad \textit{Ans. (b)}$$

The center of pressure of the total resultant force on the right side of the tank lies 13.95 ft below the surface.