University of Babylon

College of Materials Engineering

Department of Engineering of Polymer and Petrochemical Industries

Strength of Materials

B.Sc. Course for Second stage

By

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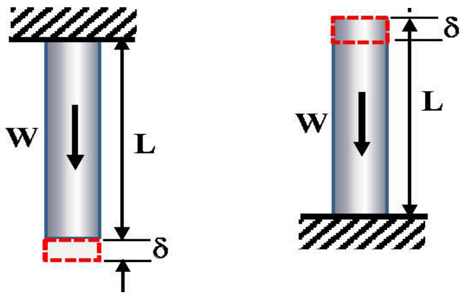
Lecture 3: Simple Strain

Strain:

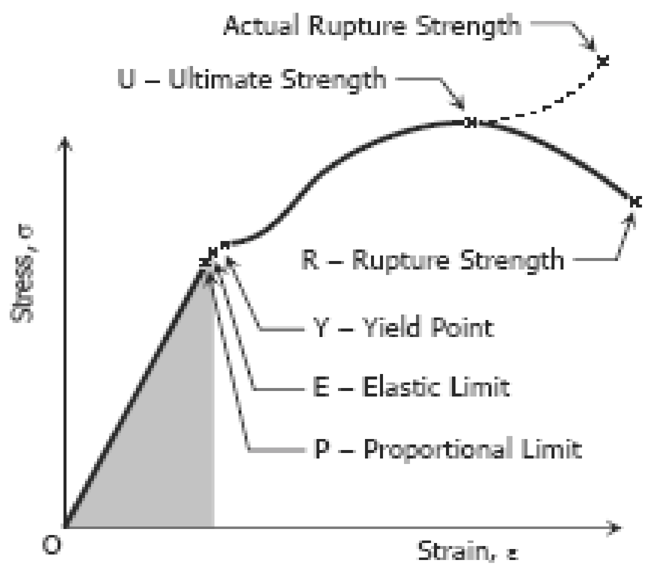
Also known as unit deformation, strain is the ratio of the change in length caused by the applied force, to the original length. $\varepsilon = \frac{\delta}{L}$



where δ is the deformation and L is the original length, thus ε is dimensionless.

Stress-Strain Diagram

Suppose that a specimen be placed in tension-compression testing machine. As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each increment of the load and this is continued until failure of the specimen takes place. Knowing the original cross-sectional area and length of the specimen, the normal stress σ and the strain ε can be obtained. The graph of these quantities with the stress σ along the y-axis and the strain ε along the x-axis is called the stress-strain diagram. The stress-strain diagram differs in form for various materials. The diagram shown below is that for a medium carbon structural steel.



PROPORTIONAL LIMIT (HOOKE'S LAW)

From the origin to the point called proportional limit, the stress-strain curve is a straight line. This linear relation between elongation and the axial force causing called Hooke's Law that within the proportional limit, the stress is directly proportional to strain or

$$\boldsymbol{\sigma} \propto \boldsymbol{\varepsilon} \ \ \boldsymbol{or} \ \ \boldsymbol{\sigma} = \boldsymbol{k\varepsilon}$$

The constant of proportionality ***k*** is called the Modulus of Elasticity ***E*** or Young's Modulus and is equal to the slope of the stress-strain diagram from origin to the proportional limit. Then

$$\boldsymbol{\sigma} = \boldsymbol{E\varepsilon}$$

ELASTIC LIMIT

The elastic limit is the limit beyond which the material will no longer go back to its original shape when the load is removed, or it is the maximum stress that may developed such that there is no permanent or residual deformation when the load is entirely removed.

ELASTIC AND PLASTIC RANGES

The region in stress-strain diagram from origin to proportional limit is called the elastic range. The region from proportional limit to fracture stress is called the plastic range.

YIELD POINT

Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.

ULTIMATE STRENGTH

The maximum ordinate in the stress-strain diagram is the ultimate strength or tensile strength.

FRACTURE STRENGTH

Fracture strength is the strength of the material at rupture. This is also known as the breaking strength.

AXIAL DEFORMATION

In the linear portion of the stress-strain diagram, the tress is proportional to strain and is given by:

$$\boldsymbol{\sigma = E\varepsilon}$$

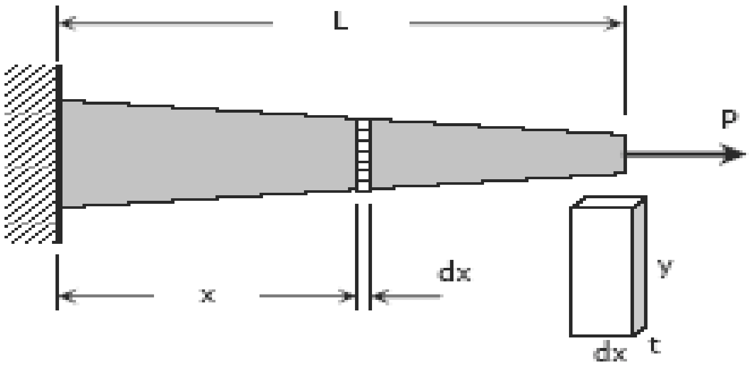
Since $\sigma = P / A$ and $\varepsilon = \delta / L$, then $P / A = E\,\delta / L$. Solving for δ,

$$\boldsymbol{\delta = \frac{PL}{AE} = \frac{\sigma L}{E}}$$

To use this formula,

1- the load must be axial,
2- the bar must have a uniform cross-sectional area, and
3- the stress must not exceed the proportional limit.

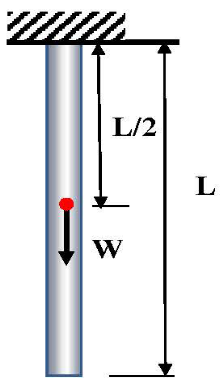
If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.



$$\delta = \frac{P}{E}\int_0^L \frac{dx}{A}$$

For a rod of unit mass ρ suspended vertically from one end, the total elongation due to its own weight is:

$$\delta = \frac{\rho g L^2}{2E} = \frac{mgL}{2AE}$$



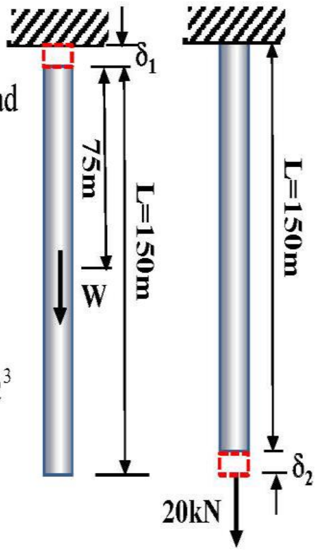
Where ρ is in kg/m^3, L is the length of the rod in mm, m is the total mass of the rod in kg, A is the cross-sectional area of the rod in mm^2, and g = 9.81 m/s^2.

STIFFNESS, k

Stiffness is the ratio of the steady force acting on an elastic body to the resulting displacement. It has the unit of N/mm.

$$k = P/\delta$$

EX: A steel rod having a cross-sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m^3 and E = 200 × 10^3 MN/m^2, find the total elongation of the rod.



Solution:

total elongation = elongation due to its own weight + elongation due to applied load

$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \frac{PL}{AE}$$

$$P = W = selfweight\ of\ rod = volume \times unit\ weight$$

$$unit\ weight = unit\ mass * g = 7850 kg/m^3 * 9.807 m/s^2 = 76984.95 N/m^3$$

$$\therefore P = volume \times unit\ weight = (A * L) \times unit\ weight$$

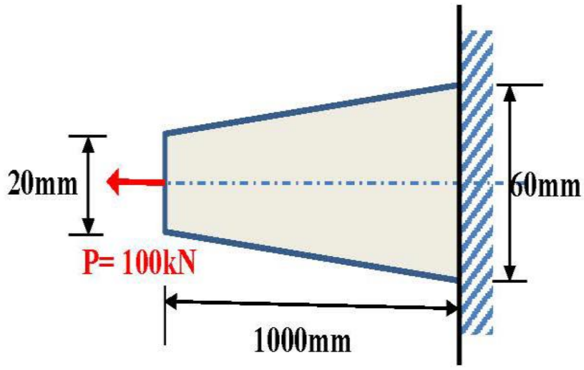
$$P = [(300 * 10^{-6}) * 150] \times 76984.95 = 3464.322 N$$

$$\delta_1 = \frac{3464.322 N * 75 m}{(300 * 10^{-6} m^2)(200000 * 10^6 N/m^2)} = 0.00433 m = 4.33 mm$$

$$\delta_2 = \frac{PL}{AE} = \frac{(20 * 10^3 N) * 150 m}{(300 * 10^{-6} m^2)(200000 * 10^6 N/m^2)} = 0.05 m = 50 mm$$

$$\therefore \delta = \delta_1 + \delta_2 = 4.33 + 50 = 54.33 \text{ mm}$$

EX: Tapered bar with thickness (t=20 mm) as shown in fig. having E=200x10^3 MPa. Determine the total deformation due applied load P=100kN.

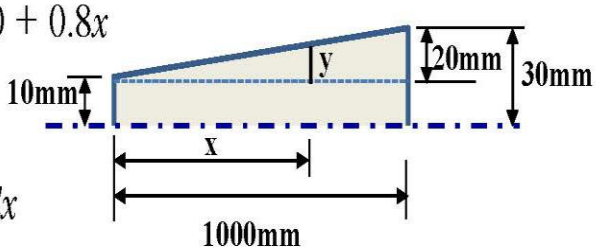


Solution:

$$\delta = \int_{x=0}^{x=L} \frac{P}{EA_x} dx$$

$$\frac{20}{1000} = \frac{y}{x} \Longrightarrow y = \frac{2}{100} x \Longrightarrow y = 0.02x$$

$$A_x = (20 + 2y) * 20 = [20 + 2 * (0.02x)] * 20 = 400 + 0.8x$$



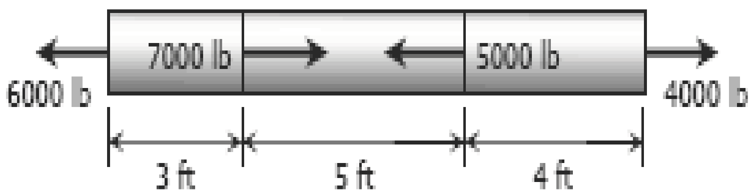
$$\delta = \int_{x=0}^{x=L} \frac{P}{EA_x} dx = \int_{x=0}^{x=1000} \frac{(100 * 10^3)}{(200 * 10^3)(400 + 0.8x)} dx$$

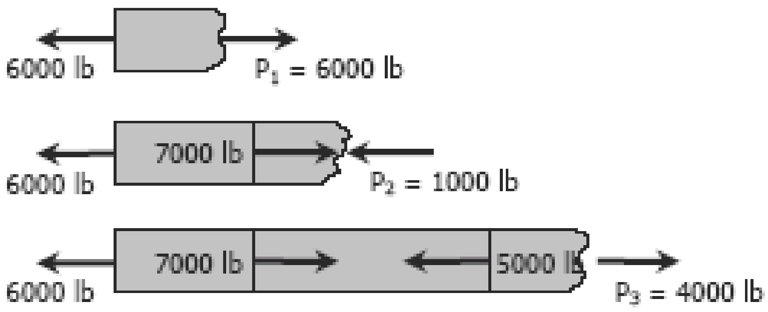
$$\delta = \frac{1}{2} \int_{0}^{1000} \frac{dx}{400 + 0.8x} \left(\frac{0.8}{0.8}\right) = \frac{1}{2} \frac{1}{0.8} [\ln(400 + 0.8x)]_0^{1000}$$

$$\delta = \frac{1}{1.6} [\ln(400 + 800) - \ln(400 + 0)] = \frac{1}{1.6} [\ln(1200) - \ln(400)]$$

$$\delta = \frac{1}{1.6} \ln \frac{1200}{400} = \frac{1}{1.6} \ln 3 = \frac{1}{1.6} (1.0986) = 0.6866 mm$$

EX: An aluminum bar having a cross-sectional area of 0.5 in^2 carries the axial loads applied at the positions shown in Fig. Compute the total change in length of the bar if E = 10 × 10^6 psi.





$P_1 = 6000$ lb tension
$P_2 = 1000$ lb compression
$P_3 = 4000$ lb tension

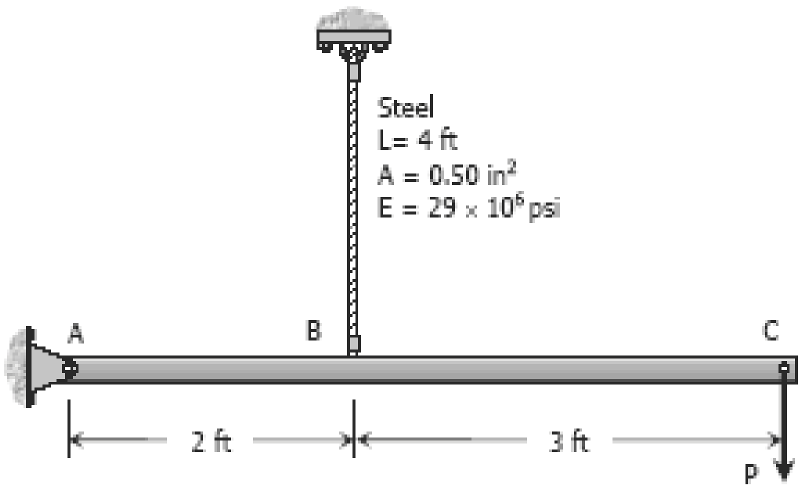
$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_1 - \delta_2 + \delta_3$$

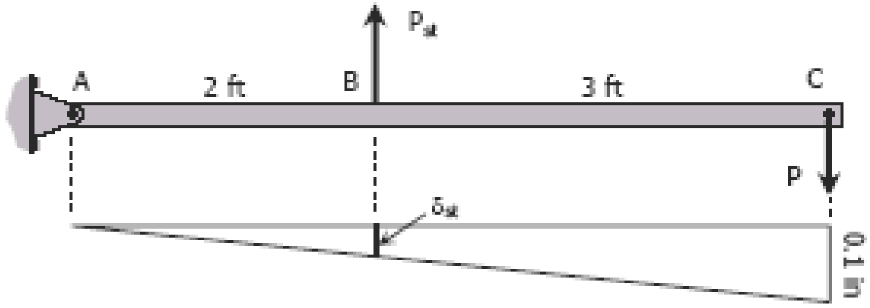
$$\delta = \frac{6000(3\times 12)}{0.5(10\times 10^6)} - \frac{1000(5\times 12)}{0.5(10\times 10^6)} + \frac{4000(4\times 12)}{0.5(10\times 10^6)}$$

$\delta = \mathbf{0.0696}$ **in (lengthening)**

EX: The rigid bar ABC shown in Fig., is hinged at A and supported by a steel rod at B. Determine the largest load P that can be applied at C if the stress in the steel rod is limited to 30 ksi and the vertical movement of end C must not exceed 0.10 in.



Free body and deformation diagrams:



Based on maximum stress of steel rod:

$\Sigma M_A = 0$

$5P = 2P_{st}$

$P = 0.4P_{st}$

$P = 0.4\sigma_{st}A_{st}$

$P = 0.4[30(0.50)]$

$P = 6$ kips

Based on movement at C:

$$\frac{\delta_{st}}{2} = \frac{0.1}{5}$$

$\delta_{st} = 0.04$ in

$$\frac{P_{st}L}{AE} = 0.04$$

$$\frac{P_{st}(4\times 12)}{0.50(29\times 10^6)} = 0.04$$

$P_{st} = 12\ 083.33$ lb

$\Sigma M_A = 0$

$5P = 2P_{st}$

$P = 0.4P_{st}$

$P = 0.4(12\ 083.33)$

$P = 4833.33$ lb $= 4.83$ kips

Use the smaller value, P = **4.83 kips**

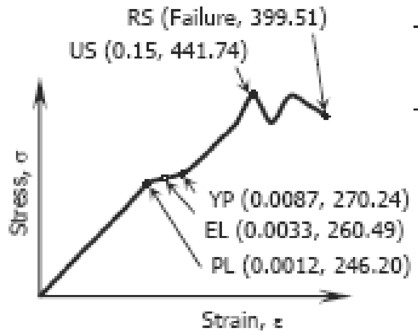
EX: The following data were recorded during the tensile test of a 14-mm-diameter mild steel rod. The gage length was 50 mm.

Load (N)	Elongation (mm)	Load (N)	Elongation (mm)
0	0	46200	1.25
6310	0.010	52400	2.50
12600	0.020	58500	4.50
18800	0.030	68000	7.50
25100	0.040	59000	12.50
31300	0.050	67800	15.50
37900	0.060	65000	20.00
40100	0.163	61500	Fracture
41600	0.433		

Plot the stress-strain diagram and determine the following mechanical properties: (a) proportional limits; (b) modulus of elasticity; (c) yield point; (d) ultimate strength; and (e) rupture strength.

Area, $A = \frac{1}{4}\pi(14)^2 = 49\pi$ mm²; Length, $L = 50$ mm

Strain = Elongation/Length; Stress = Load/Area



Stress-Strain Diagram
(not drawn to scale)

PL = Proportional Limit
EL = Elastic Limit
YP = Yield Point
US = Ultimate Strength
RS = Rupture Strength

Load (N)	Elongation (mm)	Strain (mm/mm)	Stress (MPa)
0	0	0	0
6310	0.010	0.0002	40.99
12600	0.020	0.0004	81.85
18800	0.030	0.0006	122.13
25100	0.040	0.0008	163.05
31300	0.050	0.001	203.33
37900	0.060	0.0012	246.20
40100	0.163	0.0033	260.49
41600	0.433	0.0087	270.24
46200	1.250	0.025	300.12
52400	2.500	0.05	340.40
58500	4.500	0.09	380.02
68000	7.500	0.15	441.74
59000	12.500	0.25	383.27
67800	15.500	0.31	440.44
65000	20.000	0.4	422.25
61500	Failure		399.51

From stress-strain diagram:

(a) Proportional Limit = **246.20 MPa**

(b) Modulus of Elasticity

E = slope of stress-strain diagram within proportional limit

$$E = \frac{246.20}{0.0012} = 205\,166.67 \text{ MPa}$$

$E = 205.2$ GPa

(c) Yield Point = **270.24 MPa**

(d) Ultimate Strength = **441.74 MPa**

(e) Rupture Strength = **399.51 MPa**