

**1)) NWCM:**

This method starts at the north west (upper left) corner cell of the tableau (variable  $x_{11}$ ).

**Step 1:** Allocate as much as possible to the selected cell, and adjust the associated amounts of capacity (supply) and requirement (demand) by subtracting the allocated amount.

**Step 2:** Cross out the row (column) with zero supply or demand to indicate that no further assignments can be made in that row (column). If both the row and column becomes zero in the same time, cross out one of them only, and leave a zero supply or demand in the uncrossed out row (column).

**Step 3:** If exactly one row (column) is left uncrossed out, then stop. Otherwise, move to the cell to the right if a column has just been crossed or the one below if a row has been crossed out. Go to step 1.

**Refer to above example:**

	Retail Agency					
Factories	1	2	3	4	5	Capacity
1	1	9	13	36	51	50
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Requirement	100	60	50	50	40	300

Retail Agency \ Factories	1	2	3	4	5	Capacity

1	50	1	9	13	36	51	<del>50</del>
2	50	24	50	12	16	20	1
3		14	10	33	50	1	23
							26
<b>Requirement</b>	<del>100</del>	<del>50</del>	<del>60</del>	<del>10</del>	<del>50</del>	<del>50</del>	<del>40</del>

The arrows show the order in which the allocated amounts are generated.

The starting basic solution is given as

$$x_{11} = 50, \quad x_{21} = 50, \quad x_{22} = 50, \quad x_{32} = 10, \quad x_{33} = 50, \quad x_{34} = 50, \quad x_{35} = 40.$$

The corresponding transportation cost is:

$$Z = 50 * 1 + 50 * 24 + 50 * 12 + 10 * 33 + 50 * 1 + 50 * 23 + 40 * 26 = 4420$$

It is clear that as soon as a value of  $x_{ij}$  is determined, a row (column) is eliminated from further consideration. The last value of  $x_{ij}$  eliminates both a row and column. Hence a feasible solution computed by North West Corner Method can have at most  $m + n - 1$  positive  $x_{ij}$  if the transportation problem has  $m$  sources and  $n$  destinations.

## 2)) LCM

Least cost method is also known as matrix minimum method in the sense we look for the row and the column corresponding to which  $C_{ij}$  is minimum. This method finds a better initial basic feasible solution by concentrating on the cheapest routes. Instead of starting the allocation with the northwest cell as in the North West Corner Method, we start by allocating as much as possible to the cell with the smallest unit cost. If there are two or more minimum costs then we should select the row and the column corresponding

to the lower numbered row. If they appear in the same row we should select the lower numbered column. We then cross out the satisfied row or column, and adjust the amounts of capacity and requirement accordingly. If both a row and a column is satisfied simultaneously, only one is crossed out. Next, we look for the uncrossed-out cell with the smallest unit cost and repeat the process until we are left at the end with exactly one uncrossed-out row or column.

Retail Agency Factories	1	2	3	4	5	Capacity
1	1 50	9 X	13 X	36 X	51 X	50
2	24 X	12 60	16 X	20 X	1 40	100
3	14 50	33 X	1 50	23 50	26 X	150
Requirement	100	60	50	50	40	

We observe that  $C_{11}=1$  is the minimum unit cost in the table. Hence  $X_{11}=50$  and the first row is crossed out since the row has no more capacity. Then the minimum unit cost in the uncrossed-out row and column is  $C_{25}=1$ , hence  $X_{25}=40$  and the fifth column is crossed out. Next  $C_{33}=1$  is the minimum unit cost, hence  $X_{33}=50$  and the third column is crossed out. Next  $C_{22}=12$  is the minimum unit cost, hence  $X_{22}=60$  and the second column is crossed out. Next we look for the uncrossed-out row and column now  $C_{31}=14$  is the minimum unit cost, hence  $X_{31}=50$  and crossed out the first column since it

was satisfied. Finally  $C_{34}=23$  is the minimum unit cost, hence  $X_{34}=50$  and the fourth column is crossed out.

So that the basic feasible solution developed by the Least Cost Method has transportation cost is

$$Z = 1 * 50 + 12 * 60 + 1 * 40 + 14 * 50 + 1 * 50 + 23 * 50 = 2710$$

### 3)) Vogel Approximation Method ( VAM )

**Step 1:** For each row and column find the difference between the two lowest unit shipping costs.

**Step 2:** Assign as many units as possible to the lowest-cost square in the row and column selected.

**Step 3:** Eliminate the column or row that has been satisfied.

**Example:** Consider the following transportation problem

Origin	Destination				$a_i$
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
$b_j$	60	40	30	110	240

**Solution:**

1. Compute the penalty for various rows and columns.
2. Look for the highest penalty in the row or column, the highest penalty occurs in the second column and the minimum unit cost i.e.  $c_{ij}$  in this column is  $c_{12}=22$ . Hence assign 40 to this cell i.e.  $x_{12}=40$  and cross out the second column (since second column was satisfied ).

Origin	Destination				$a_i$	column Penalty
	1	2	3	4		

1	20 <b>X</b>	22 40	17 <b>X</b>	4 80	120	<b>13</b>
2	24	37 <b>X</b>	9 30	7	70	<b>2</b>
3	32	37 <b>X</b>	20 <b>X</b>	15	50	<b>5</b>
$b_j$ Row Penalty	60 <b>4</b>	40 <b>15</b>	30 <b>8</b>	110 <b>3</b>	240	

Origin	Destination				$a_i$	column Penalty
	1	2	3	4		
1	20 <b>X</b>	22 40	17 <b>X</b>	4 80	120	<b>13</b>
2	24 10	37 <b>X</b>	9 30	7 30	70	<b>2 17</b>
3	32 50	37 <b>X</b>	20 <b>X</b>	15 <b>X</b>	50	<b>5 17</b>
$b_j$ Row Penalty	60 <b>4</b> <b>8</b>	40 <b>15</b>	30 <b>8</b>	110 <b>3</b> <b>8</b>	240	

The transportation cost corresponding to this choice of basic variables is:

$$Z = 22 * 40 + 4 * 80 + 30 * 9 + 30 * 7 + 10 * 24 + 50 * 32 = 3520.$$