Infeasible Solution

In the next example we will illustrate how to identify the infeasible solution using simplex method.

Example

Consider the following problem.

$$\text{Minimize} \quad 200\,x_1 + 300\,x_2$$

Subject to:

$$2\,x_1 + 3\,x_2 \geq 1200$$

$$x_1 + x_2 \leq 400$$

$$2\,x_1 + \frac{3}{2}\,x_2 \geq 900$$

$$x_1, x_2 \geq 0$$

Solution

Since it is a minimization problem we have to convert it into maximization problem and introduce the slack, surplus and artificial variables.

$$\text{Minimize} \quad -200\,x_1 - 300\,x_2$$

Subject to:

$$2\,x_1 + 3\,x_2 - S_3 + a_6 = 1200$$

$$x_1 + x_2 + S_4 = 400$$

$$2\,x_1 + \frac{3}{2}\,x_2 - S_5 + a_7 = 900$$

$$x_1, x_2, S_3, S_4, S_5, a_6, a_7 \geq 0$$

Here the a_6 and a_7 are the artificial variables. We use two phase method to solve this problem.

Phase I

Maximize $-a_6 - a_7$

Subject to:

$$2x_1 + 3x_2 - S_3 + a_6 = 1200$$

$$x_1 + x_2 + S_4 = 400$$

$$2x_1 + \frac{3}{2}x_2 - S_5 + a_7 = 900$$

$$x_1, x_2, S_3, S_4, S_5, a_6, a_7 \geq 0$$

The calculation of simplex procedures and tables are as follows:

CB	Basic variables	C_j XB	0 x_1	0 x_2	0 s_3	0 s_4	0 s_5	-1 a_6	-1 a_7
-1	a_6	1200	2	3	-1	0	0	1	0
0	s_4	400	1	1	0	1	0	0	1
-1	a_7	900	2	3/2	0	0	-1	0	0
		z_j-c_j	-4	-9/2	1	0	1	0	0

CB	Basic variables	C_j XB	0 x_1	0 x_2	0 s_3	0 s_4	0 s_5	-1 a_7
0	x_2	400	2/3	1	-1/3	0	0	0
0	s_4	0	1/3	0	1/3	1	0	0
-1	a_7	300	1	0	1/2	0	-1	1
		z_j-c_j	-1	0	-1/2	0	1	0

CB	Basic variables	C_j XB	0 x_1	0 x_2	0 s_3	0 s_4	0 s_5	-1 a_7
0	x_2	400	0	1	-1	-2	0	0
0	x_1	0	1	0	1	3	0	0
-1	a_7	300	0	0	-1/2	-3	-1	1
		z_j-c_j	0	0	1/2	3	1	0

Note that $z_j - c_j \geq 0$ for all the variables but the artificial variable a_7 is still a basic variable. This situation indicates that the problem has no feasible solution.

Exercises:

Q1. A soft drinks company has a two products viz. Coco-cola and Pepsi with profit of $2 and $1 per unit. The following table illustrates the labour, equipment and materials to produce per unit of each product. Determine suitable product mix which maximizes the profit using simplex method.

	Pepsi	Coco-cola	Total Resources
Labour	3	2	12
Equipment	1	2.3	6.9
Material	1	1.4	4.9

Q2. Solve the following linear programming problem using two phase and M method.

Maximize

$$12x_1 + 15x_2 + 9x_3$$

Subject to:

$$8x_1 + 16x_2 + 12x_3 \leq 250$$
$$4x_1 + 8x_2 + 10x_3 \geq 80$$
$$7x_1 + 9x_2 + 8x_3 = 105$$
$$x_1, x_2, x_3 \geq 0$$

Q3. Solve the following linear programming problem using simplex method.

Maximize

$$3x_1 + 2x_2$$

Subject to:

$$x_1 - x_2 \leq 1$$
$$x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

Q4. Solve the following linear programming problem using simplex method.

Maximize

$$x_1 + x_2$$

Subject to:

$$-2x_1 + x_2 \leq 1$$
$$x_1 \leq 2$$
$$x_1 + x_2 \leq 3$$
$$x_1, x_2, x_3 \geq 0$$

Q5. Solve the following linear programming problem using simplex method.

Maximize

$$P = 3x_1 + 4x_2 + x_3$$

Subject to:

$$x_1 + 2x_2 + x_3 \leq 6$$
$$2x_1 + 2x_3 \leq 4$$
$$3x_1 + x_2 + x_3 \leq 9$$
$$x_1, x_2, x_3 \geq 0$$

Q_1. Coco-Cola = 20/9, Pepsi = 161/90

Maximum Profit = $ 6.23

Q_2. $x_1 = 6$, $x_2 = 7$, $x_3 = 0$

Maximum Profit = 177

Q_3. Unbounded Solution

Q_4. $x_1 = 2$, $x_2 = 1$ or $x_1 = 2/3$, $x_2 = 7/3$

Maximum Profit = 3.

Q5. $x_1 = 2$, $x_2 = 2$, $x_3 = 0$

Maximum P = 14