## 2.3 A context-free grammar (CFG):

A production rules of the grammar have the form **α → β,** each production in P satisfies **|α|= 1**; i.e., α is a single nonterminal.

- A language generated from a context-free grammar is called a context-free language. Any context-free language is context sensitive.
- The grammars are called context free because – since all rules only have a nonterminal on the left hand side – one can always replace that nonterminal symbol with what is on the right hand side of the rule.
- The automata that recognizes context-free languages is a push-down automaton.

***Example 1:*** $\{a^n b^n c^n \mid n \geq 0\}$ is context-sensitive but not context-free.

Here is a csg.

$$S \to \Lambda \mid abc \mid aTBc$$
$$T \to abC \mid aTBC$$
$$CB \to BC$$
$$B \to b.$$
$$C \to c.$$

-Derive aaabbbccc.

S ⇒ aTBc ⇒ aaTBCBc ⇒ aaabCBCBc ⇒ aaabBCCBc ⇒ aaabbCCBc⇒aaabbCBCc ⇒ aaabbBCCc ⇒ aaabbbCCc ⇒ aaabbbCcc ⇒ aaabbbccc.

***Example 2:***

Let $L(G1) = \{0^n1^n \mid n \geq 0\}$ and $L(G2) = \{0^n\#1^n \mid n \geq 0\}$. Given two CFLs, it is easy to construct a CFG for their union, e.g., combining CFGs for L(G1) and L(G2):

$$S \to S_1 \mid S_2$$
$$S_1 \to 0S_11 \mid \epsilon$$
$$S_2 \to 0S_21 \mid \#$$

***Example 3:***

$$S \to abS$$
$$S \to a$$

L(G)=(ab)*a

## 2.4 Regular Grammar(RG):

G is a Type-3 or right-linear or regular grammar if each production has one of the following three forms: A→cB, A → c, A →ϵ; where A,B are nonterminals (with B = A allowed) and c is a terminal.

- The regular languages are a proper subset of the context-free languages.

- Such a grammar restricts its rules to a *single nonterminal* on the *left-hand side* and a right-hand side consisting of a single terminal, possibly followed (or preceded, but not both in the same grammar) by a single nonterminal.
- Regular languages can be considered as special types of context free languages, i.e. all regular languages are CF languages but not all CF languages are regular.

***Example 4***

The following grammar is unrestricted.

$$\{S \rightarrow TbC, \quad Tb \rightarrow c, \quad cC \rightarrow Sc \mid \wedge\}$$

This grammar is not context-sensitive, not context-free, and not regular. But can transform it into $S \rightarrow Sc \mid \wedge$. So the language of the grammar is regular.

Regular grammar generate regular languages as in following examples:

***Example 5:***

$$\{S \rightarrow Aab, \quad A \rightarrow Aab|B, \quad B \rightarrow a\}$$

**L(G)=aab(ab)***

***Example 6:***

The CFG ({S}, {a, b}, S, P) with P consisting of the following productions:

$$\{S \rightarrow aSb, \quad S \rightarrow \epsilon\}$$

The grammar is not regular because of the ***b*** on the right of S.

It generates the language $a^n b^n$ where $n \geq 0$. This is not a regular language but it can be generated by a context free grammar is therefore a context free language.

***Homework 1:***

$G = (\{S\}, \{0, 1\}, \{S \rightarrow 0S1|\epsilon\}, S)$

- Is $\epsilon$ in L(G)?
- Is 01 in L(G)?
- Is 0011 in L(G)?
- Is $0^n 1^n$ in L(G)?

What language is defined by the following G?

$S \rightarrow \epsilon$

$S \rightarrow 0S1$

What language is defined by the following G?

$\{S \rightarrow \epsilon, \quad S \rightarrow 0S0, \quad S \rightarrow 1S1\}$

***Exercise 1:***

What is language generated by this grammar G given by the productions

$$S \rightarrow 0S0 \mid 0B0$$
$$B \rightarrow 1B \mid 1$$