Transportation problem :
Methods for Initial Basic Feasible Solution
(North - West corner rule and matrix minimum method)

Methods for Initial Basic Feasible Solution

Some simple methods to obtain the initial basic feasible solution are

1. North-West Corner Rule
2. Lowest Cost Entry Method (Matrix Minima Method)
3. Vogel's Approximation Method (Unit Cost Penalty Method)

1- North-West Corner Rule

Step 1

- The first assignment is made in the cell occupying the upper left-hand (north-west) corner of the table.
- The maximum possible amount is allocated here i.e. $x_{11} = \min(a_1, b_1)$. This value of x_{11} is then entered in the cell (1,1) of the transportation table.

Step 2

i. If $b_1 > a_1$, move vertically downwards to the second row and make the second allocation of amount $x_{21} = \min(a_2, b_1 - x_{11})$ in the cell (2, 1).
ii. If $b_1 < a_1$, move horizontally right side to the second column and make the second allocation of amount $x_{12} = \min(a_1 - x_{11}, b_2)$ in the cell (1, 2).
iii. If $b_1 = a_1$, there is tie for the second allocation. One can make a second allocation of magnitude $x_{12} = \min(a_1 - a_1, b_2)$ in the cell (1, 2) or $x_{21} = \min(a_2, b_1 - b_1)$ in the cell (2, 1)

Step 3

Start from the new north-west corner of the transportation table and repeat steps 1 and 2 until all the requirements are satisfied.

Find the initial basic feasible solution by using North-West Corner Rule

1.

W→ F↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

Solution

	W_1	W_2	W_3	W_5	Availability
F_1	5 (19)	2 (30)			7 2 0
F_2		6 (30)	3 (40)		9 3 0
F_3			4 (70)	14 (20)	18 14 0
Requirement	5 0	8 6 0	7 4 0	14 0	

Initial Basic Feasible Solution

$x_{11} = 5$, $x_{12} = 2$, $x_{22} = 6$, $x_{23} = 3$, $x_{33} = 4$, $x_{34} = 14$

The transportation cost is 5 (19) + 2 (30) + 6 (30) + 3 (40) + 4 (70) + 14 (20) = Rs. 1015

2.

	D_1	D_2	D_3	D_4	Supply
O_1	1	5	3	3	34
O_2	3	3	1	2	15
O_3	0	2	2	3	12
O_4	2	7	2	4	19
Demand	21	25	17	17	80

Solution

	D_1	D_2	D_3	D_4	Supply
O_1	21 (1)	13 (5)			34 13 0
O_2		12 (3)	3 (1)		15 3 0
O_3			12 (2)		12 0
O_4			2 (2)	17 (4)	19 17
Demand	21 0	25 12 0	17 14 2 0	17 0	

Initial Basic Feasible Solution

$x_{11} = 21$, $x_{12} = 13$, $x_{22} = 12$, $x_{23} = 3$, $x_{33} = 12$, $x_{43} = 2$, $x_{44} = 17$

The transportation cost is 21 (1) + 13 (5) + 12 (3) + 3 (1) + 12 (2) + 2 (2) + 17 (4) = Rs. 221

3.

From	To					Supply
	2	11	10	3	7	4
	1	4	7	2	1	8
	3	1	4	8	12	9
Demand	3	3	4	5	6	

Solution

From	To					Supply
	3 (2)	1 (11)				4 1 0
		2 (4)	4 (7)	2 (2)		8 6 2 0
				3 (8)	6 (12)	9 6 0
Demand	3 0	3 2 0	4 0	5 3 0	6 0	

Initial Basic Feasible Solution

$x_{11} = 3$, $x_{12} = 1$, $x_{22} = 2$, $x_{23} = 4$, $x_{24} = 2$, $x_{34} = 3$, $x_{35} = 6$

The transportation cost is 3 (2) + 1 (11) + 2 (4) + 4 (7) + 2 (2) + 3 (8) + 6 (12) = Rs. 153

2 - **Lowest Cost Entry Method (Matrix Minima Method)**

Step 1

Determine the smallest cost in the cost matrix of the transportation table. Allocate x_{ij} = min (a_i, b_j) in the cell (i, j)

Step 2

- If $x_{ij} = a_i$, cross out the i^{th} row of the table and decrease b_j by a_i. Go to step 3.
- If $x_{ij} = b_j$, cross out the j^{th} column of the table and decrease a_i by b_j. Go to step 3.
- If $x_{ij} = a_i = b_j$, cross out the i^{th} row or j^{th} column but not both.

Step 3

Repeat steps 1 and 2 for the resulting reduced transportation table until all the requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

Find the initial basic feasible solution using Matrix Minima method

1.

	W_1	W_2	W_3	W_4	Availability
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	

Solution

	W_1	W_2	W_3	W_4	
F_1	(19)	(30)	(50)	(10)	7
F_2	(70)	(30)	(40)	(60)	9
F_3	(40)	8 (8)	(70)	(20)	10
	5	X	7	14	

	W_1	W_2	W_3	W_4	
F_1	(19)	(30)	(50)	7 (10)	X
F_2	(70)	(30)	(40)	(60)	9
F_3	(40)	8 (8)	(70)	(20)	10
	5	X	7	7	

	W_1	W_2	W_3	W_4	
F_1	(19)	(30)	(50)	7 (10)	X
F_2	(70)	(30)	(40)	(60)	9
F_3	(40)	8 (8)	(70)	7 (20)	3
	5	X	7	X	

	W_1	W_2	W_3	W_4	
F_1	(19)	(30)	(50)	7 (10)	X
F_2	(70)	(30)	(40)	(60)	9
F_3	3 (40)	8 (8)	(70)	7 (20)	X
	2	X	7	X	

	W_1	W_2	W_3	W_4	
F_1	(19)	(30)	(50)	7 (10)	X
F_2	2 (70)	(30)	7 (40)	(60)	X
F_3	3 (40)	8 (8)	(70)	7 (20)	X
	X	X	X	X	

Initial Basic Feasible Solution

$x_{14} = 7$, $x_{21} = 2$, $x_{23} = 7$, $x_{31} = 3$, $x_{32} = 8$, $x_{34} = 7$

The transportation cost is 7 (10) + 2 (70) + 7 (40) + 3 (40) + 8 (8) + 7 (20) = Rs. 814

2.

	To					Availability
	2	11	10	3	7	4
From	1	4	7	2	1	8
	3	9	4	8	12	9
Requirement	3	3	4	5	6	

Solution

To

				4 (3)		4 0
From	3 (1)				5 (1)	8 5 0
		3 (9)	4 (4)	1 (8)	1 (12)	9 5 4 1 0
	3 0	3 0	4 0	5 1 0	6 1 0	

Initial Basic Feasible Solution

$x_{14} = 4$, $x_{21} = 3$, $x_{25} = 5$, $x_{32} = 3$, $x_{33} = 4$, $x_{34} = 1$, $x_{35} = 1$

The transportation cost is 4 (3) + 3 (1) + 5(1) + 3 (9) + 4 (4) + 1 (8) + 1 (12) = Rs. 78