Lecture 4

Linear Programming :

Special Cases in Graphical Method

4.1 Multiple Optimal Solution

Example 1

Solve by using graphical method

Max $Z = 4x_1 + 3x_2$

Subject to

$4x_1 + 3x_2 \leq 24$

$x_1 \leq 4.5$

$x_2 \leq 6$

$x_1 \geq 0$, $x_2 \geq 0$

Solution

The first constraint $4x_1 + 3x_2 \leq 24$, written in a form of equation

$4x_1 + 3x_2 = 24$

Put $x_1 = 0$, then $x_2 = 8$

Put $x_2 = 0$, then $x_1 = 6$

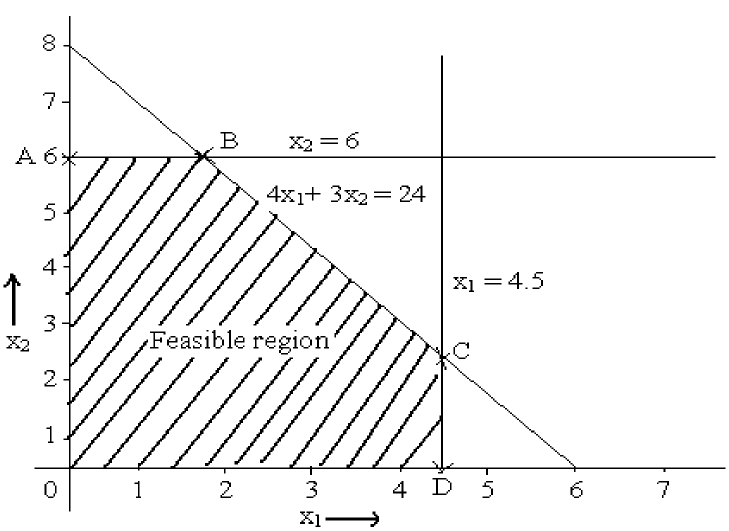
The coordinates are (0, 8) and (6, 0)

The second constraint $x_1 \leq 4.5$, written in a form of equation

$x_1 = 4.5$

The third constraint $x_2 \leq 6$, written in a form of equation

$x_2 = 6$



The corner points of feasible region are A, B, C and D. So the coordinates for the corner points are
A (0, 6)
B (1.5, 6) (Solve the two equations $4x_1 + 3x_2 = 24$ and $x_2 = 6$ to get the coordinates)
C (4.5, 2) (Solve the two equations $4x_1 + 3x_2 = 24$ and $x_1 = 4.5$ to get the coordinates)
D (4.5, 0)

We know that Max $Z = 4x_1 + 3x_2$
At A (0, 6)
$Z = 4(0) + 3(6) = 18$

At B (1.5, 6)
$Z = 4(1.5) + 3(6) = 24$

At C (4.5, 2)
$Z = 4(4.5) + 3(2) = 24$

At D (4.5, 0)
$Z = 4(4.5) + 3(0) = 18$

Max Z = 24, which is achieved at both B and C corner points. It can be achieved not only at B and C but every point between B and C. Hence the given problem has multiple optimal solutions.

4.2 No Optimal Solution

Example 1

Solve graphically

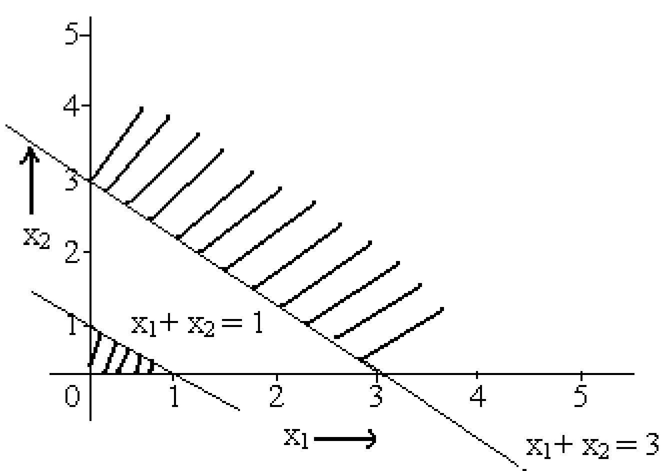
Max Z = $3x_1 + 2x_2$
Subject to

$x_1 + x_2 \leq 1$
$x_1 + x_2 \geq 3$
$x_1 \geq 0$, $x_2 \geq 0$

Solution

The first constraint $x_1 + x_2 \leq 1$, written in a form of equation
$x_1 + x_2 = 1$
Put $x_1 = 0$, then $x_2 = 1$
Put $x_2 = 0$, then $x_1 = 1$
The coordinates are (0, 1) and (1, 0)

The first constraint $x_1 + x_2 \geq 3$, written in a form of equation
$x_1 + x_2 = 3$
Put $x_1 = 0$, then $x_2 = 3$
Put $x_2 = 0$, then $x_1 = 3$
The coordinates are (0, 3) and (3, 0)



There is no common feasible region generated by two constraints together i.e. we cannot identify even a single point satisfying the constraints. Hence there is no optimal solution.

4. 3 Unbounded Solution

Example

Solve by graphical method

Max $Z = 3x_1 + 5x_2$

Subject to

$2x_1 + x_2 \geq 7$

$x_1 + x_2 \geq 6$

$x_1 + 3x_2 \geq 9$

$x_1 \geq 0$, $x_2 \geq 0$

Solution

The first constraint $2x_1 + x_2 \geq 7$, written in a form of equation

$2x_1 + x_2 = 7$

Put $x_1 = 0$, then $x_2 = 7$

Put $x_2 = 0$, then $x_1 = 3.5$

The coordinates are (0, 7) and (3.5, 0)

The second constraint $x_1 + x_2 \geq 6$, written in a form of equation

$x_1 + x_2 = 6$

Put $x_1 = 0$, then $x_2 = 6$

Put $x_2 = 0$, then $x_1 = 6$

The coordinates are (0, 6) and (6, 0)

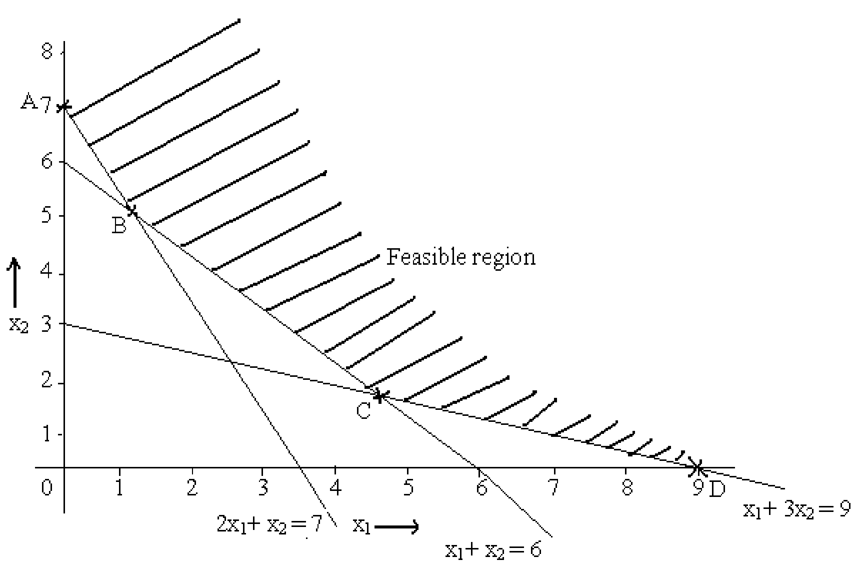
The third constraint $x_1 + 3x_2 \geq 9$, written in a form of equation

$x_1 + 3x_2 = 9$

Put $x_1 = 0$, then $x_2 = 3$

Put $x_2 = 0$, then $x_1 = 9$

The coordinates are (0, 3) and (9, 0)



The corner points of feasible region are A, B, C and D. So the coordinates for the corner points are

A (0, 7)

B (1, 5) (Solve the two equations $2x_1 + x_2 = 7$ and $x_1 + x_2 = 6$ to get the coordinates)

C (4.5, 1.5) (Solve the two equations $x_1 + x_2 = 6$ and $x_1 + 3x_2 = 9$ to get the coordinates)

D (9, 0)

We know that Max $Z = 3x_1 + 5x_2$

At A (0, 7)

$Z = 3(0) + 5(7) = 35$

At B (1, 5)

$Z = 3(1) + 5(5) = 28$

At C (4.5, 1.5)

$Z = 3(4.5) + 5(1.5) = 21$

At D (9, 0)

$Z = 3(9) + 5(0) = 27$

The values of objective function at corner points are 35, 28, 21 and 27. But there exists infinite number of points in the feasible region which is unbounded. The value of objective function will be more than the value of these four corner points i.e. the maximum value of the objective function occurs at a point at ∞. Hence the given problem has unbounded solution.