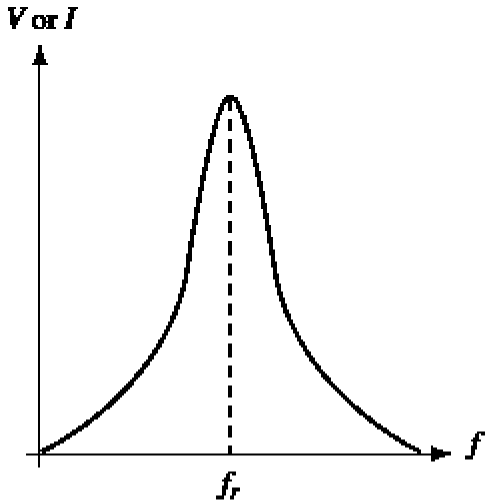
Resonance

The resonant(or tuned) circuit, in one of its many forms, allows us to select a desired radio or television signal from the vast number of signals that are around us at any time. Resonant electronic circuits contain at least one inductor and one capacitor and have a bell-shaped response curve centered at some resonant frequency, f_r, as illustrated in Figure below.

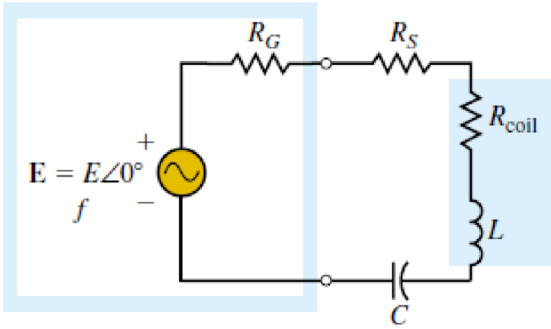


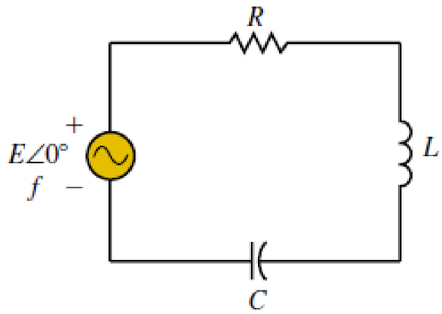
The response curve of Figure indicates that current or voltage will be at a maximum at the resonant frequency, f_r. Varying the frequency in either direction results in a reduction of the voltage or current.

If we were to apply variable-frequency sinusoidal signals to a circuit consisting of an inductor and capacitor, we would find that maximum energy will transfer back and forth between the two elements at the resonant frequency.

There are two types of resonant circuits: <u>series</u> and <u>parallel</u>. Each will be considered in some detail in this lecture.

Series Resonance:





In this circuit, the total resistance is expressed as

$$R = R_G + R_S + R_{Coil}$$

Where:

R_G: generator resistance

R_S: Series resistance

R_{Coil}: resistance of inductance coil

The total impedance of this network at any frequency is determined By

$$Z_{T_S} = R + jX_L - jX_C = R + j(X_L - X_C)$$

Resonance results when the imaginary part is zero or

$X_L = X_C$

The total impedance at resonance is

$$Z_{T_S} = R$$

The subscript s will be employed to indicate series resonant conditions. The resonant frequency can be determined in terms of the inductance and capacitance by examining the defining equation for resonance.

$X_L = X_C$

$$\omega L = \frac{1}{\omega C}$$

The series resonance frequency, ωs (in radians per second) as follows:

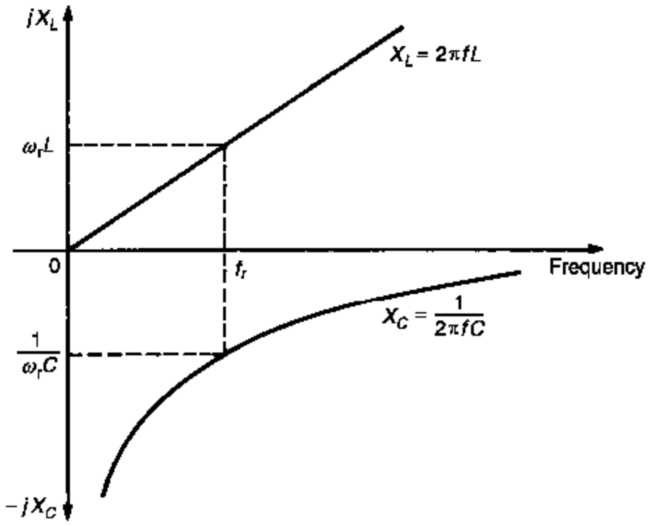
$$\omega_s = \frac{1}{\sqrt{LC}} \qquad (\frac{rad}{s})$$

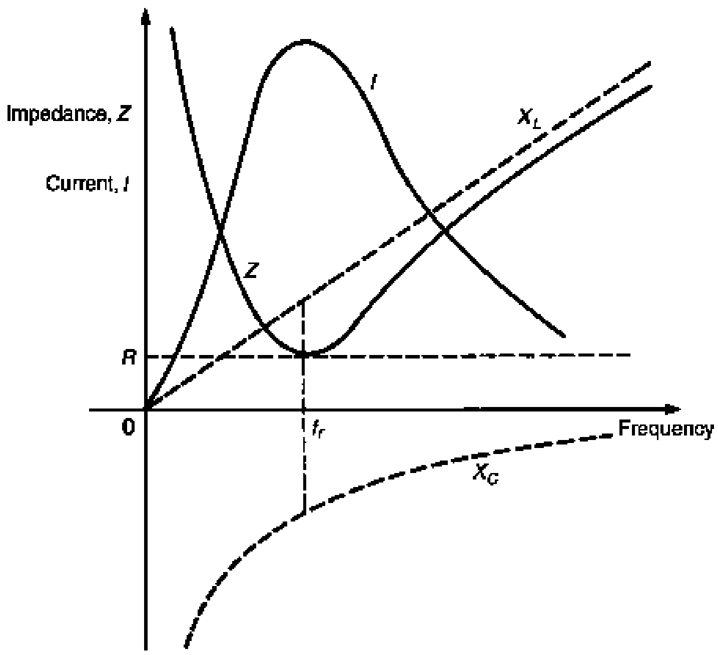
The resonance frequency in hertz as:

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad (Hz)$$

At resonance, impedance Z is a minimum value the current has maximum value, the total current in the circuit is determined from Ohm's law as

$$I = \frac{E}{Z_T} = \frac{E\angle 0^0}{R\angle 0^0} = \frac{E}{R}\angle 0^0$$



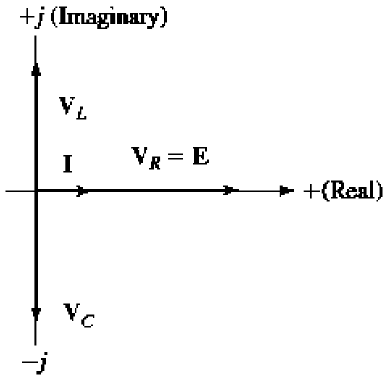


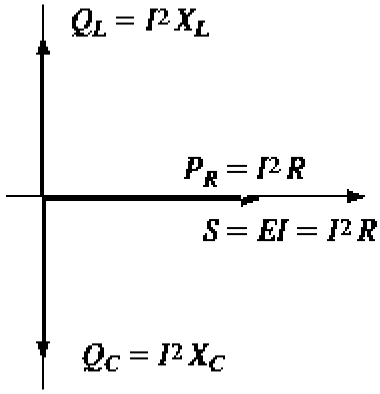
By again applying Ohm's law, , Since the current is the same through the capacitor and inductor, the voltage across each is equal in magnitude but 180° out of phase at resonance, we find the voltage across each of the elements in the circuit as follows

$$V_R = IR\angle 0^0$$

$$V_L = IX_L\angle 90^0$$

$$V_C = IX_C\angle -90^0$$





and, since XL = XC, the magnitude of V_L equals V_C at resonance; that is,

$$V_{L_S} = V_{C_S}$$

We determine the average power dissipated P_R by the resistor and the reactive powers of the inductor Q_L and capacitor Q_C as follows:

$$P_R = I^2R \quad (W)$$

$$Q_L = I^2X_L \quad (VAR)$$

$$Q_C = I^2X_C \quad (VAR)$$

Quality factor (Q):

The ratio of reactive power to average power called quality factor Q:

$$Q_S = \frac{reactive\ power}{average\ power}$$

$$Q_S = \frac{I^2X_L}{I^2R}$$

$$Q_S = \frac{X_L}{R} = \frac{\omega L}{R}$$

We now examine how the Q_S of a circuit is used in determining other quantities of the circuit. By multiplying both the numerator and denominator of Equation by the current, I, we have the following:

$$Q_S = \frac{IX_L}{IR} = \frac{V_L}{E}$$

Now, since the magnitude of the voltage across the capacitor is equal to the magnitude of the voltage across the inductor at resonance, we see that the voltages across the inductor and capacitor are related to the Q_S by the following expression:

$V_C = V_L = Q_S E$ (At resonance)

Impedance of a Series Resonant Circuit:

We examine how the impedance of a series resonant circuit varies as a function of frequency. Because the impedances of inductors and capacitors are dependent upon frequency, the total impedance of a series resonant circuit must similarly vary with frequency.

The total impedance of a simple series resonant circuit is written as

$$z_T = R + j(X_L - X_C) = R + j(\frac{\omega^2 LC - 1}{\omega C})$$

The magnitude and phase angle of the impedance vector, Z_T, are expressed as follows:

$$Z_T = \sqrt{R^2 + (\frac{\omega^2 LC - 1}{\omega C})^2}$$

$$\theta = tan^{-1}(\frac{\omega^2 LC - 1}{\omega RC})$$

When $\omega = \omega_s$

$$Z_T = R$$

$$\theta = tan^{-1} 0 = 0^0$$

When $\omega < \omega_s$

As we decrease ω from resonance, Z_T will get larger until $\omega = 0$. At this point, the magnitude of the impedance will be undefined, corresponding to an open circuit. As one might expect, the large impedance occurs because the capacitor behaves like an open circuit at D.C.

The angle θ will occur between of 0° and -90^0since the numerator of the argument of the arctangent function will always be negative, corresponding to an angle in the fourth quadrant. Because the angle of the impedance has a negative sign, we conclude that the impedance must appear capacitive in this region

When $\omega > \omega_s$

As ω is made larger than resonance, the impedance Z_T will increase due to the increasing reactance of the inductor.

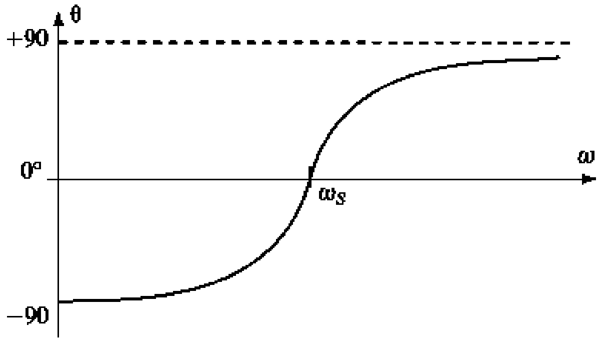
For these values of ω, the angle Ɵ will always be within 0° and 90° because both the numerator and the denominator of the arctangent function are positive. Because the angle of Z_T occurs in the first quadrant, the impedance must be inductive.

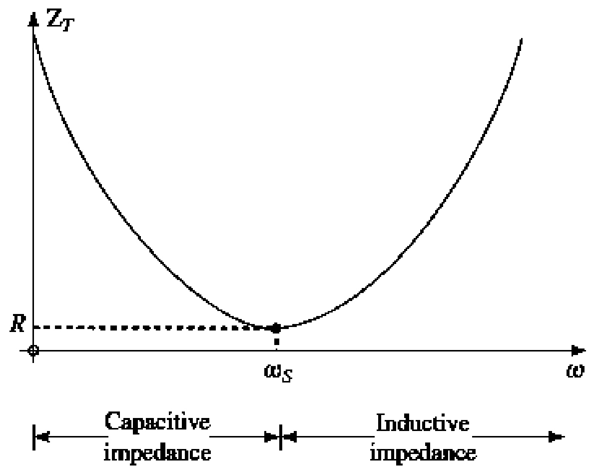
At low frequency $X_C > X_L$ and Ɵ will approach - 90^0 (cap.)

$$tan^{-1}(-X) = -tan^{-1}(X)$$

At high frequencies $X_L > X_C$ and Ɵ will approach 90^0

For the $tan^{-1}(X)$, the larger X is , the larger angle Ɵ (Closer 90^0)



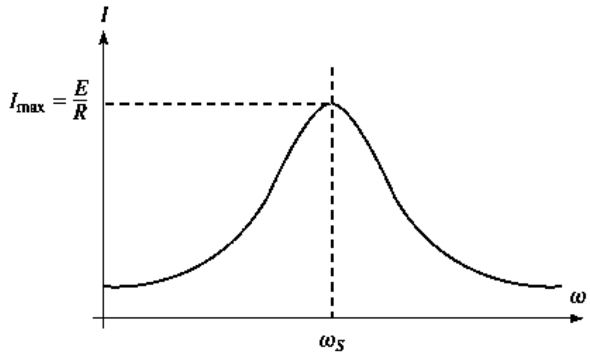


Power, Bandwidth, and Selectivity of a Series Resonant Circuit:

Due to the changing impedance of the circuit, we conclude that if a constant amplitude voltage is applied to the series resonant circuit, the current and power of the circuit will not be constant at all frequencies. In this section, we examine how current and power are affected by changing the frequency of the voltage source.

Applying Ohm's law gives the magnitude of the current as follows:

$$I_{max.} = \frac{E}{R} \quad (At\ resonance)$$



When the frequency is zero (D.C), the current will be zero since the capacitor is effectively an open circuit.

On the other hand, at increasingly higher frequencies, the inductor begins to approximate an open circuit, once again causing the current in the circuit to approach zero.

The total power dissipated by the circuit at any frequency is given as

$$P = I^2 R$$

Since the current is maximum at resonance, it follows that the power must similarly be maximum at resonance. The maximum power dissipated by the series resonant circuit is therefore given as

$$P_{max.} = I_{max.}^2 R = \frac{E^2}{R}$$

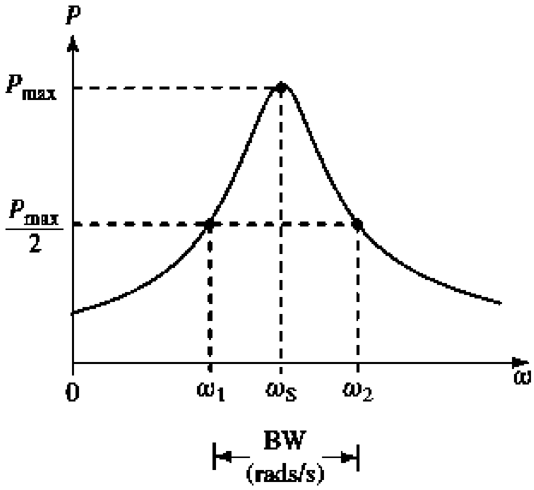
Half-power frequencies are those frequencies at which the power delivered is one-half that delivered at the resonant frequency,

$$P_{hpf} = \frac{1}{2} P_{max.}$$

And

$$P_{hpf} = I^2 R = (\frac{I_{max.}}{\sqrt{2}})^2 R = \frac{1}{2} P_{max.}$$

The power response of a series resonant circuit has a bell-shaped curve called the selectivity curve, which is similar to the current response.



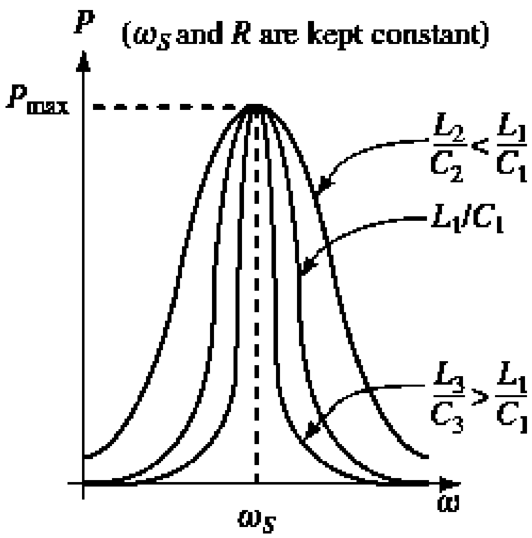
We define the bandwidth, BW, of the resonant circuit to be the difference between the frequencies at which the circuit delivers half of the maximum power. The frequencies ω_1 and ω_2 are called the *half power frequencies*, the *cutoff frequencies*, or *the band frequencies*.

If the bandwidth of a circuit is kept very narrow, the circuit is said to have a high selectivity. On the other hand, if the bandwidth of a circuit is large, the circuit is said to have a low selectivity.

The elements of a series resonant circuit determine not only the frequency at which the circuit is resonant, but also the shape (and hence the bandwidth) of the power response curve.

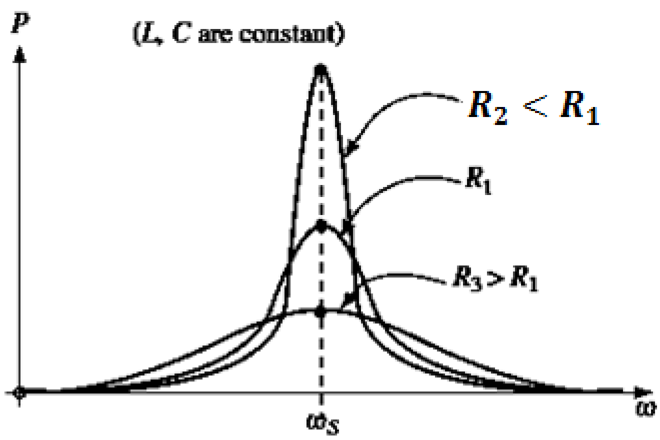
Consider a circuit in which the resistance, R, and the resonant frequency, ω_s, are held constant,

We find that by increasing the ratio of L/C, the sides of the power response curve become steeper. This in turn results in a decrease in the bandwidth. Inversely، decreasing the ratio of L/C causes the sides of the curve to become more gradual, resulting in an increased bandwidth.



If the resistance is made smaller with a fixed inductance and capacitance, the bandwidth decreases and the selectivity increases. A series circuit has the highest selectivity if the resistance of the circuit is kept to a minimum.

In terms of Qs, if R is larger for the same XL, then Qs is less, as determined by the equation ($Q_S = \frac{\omega_S L}{R}$). A small Qs, therefore, is associated with a resonant curve having a large bandwidth and a small selectivity, while a large Qs indicates the opposite.



For the series resonant circuit the power at any frequency is determined as

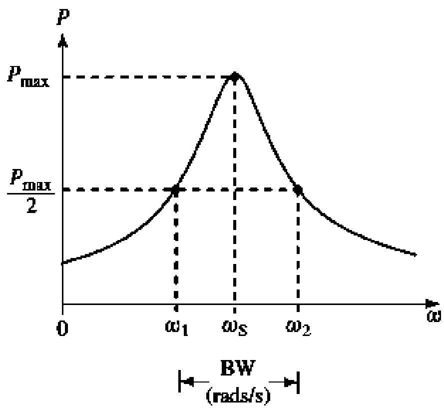
$$P = I^2 R = (\frac{E}{Z_T})^2 R = \frac{E^2\, R}{R^2 + (X_L - X_C)^2}$$

At the half-power frequencies, the power must be

$$P_{hpf} = \frac{E^2}{2R}$$

The cutoff frequencies are found by evaluating the frequencies at which the power dissipated by the circuit is half of the maximum power

$$\frac{E^2}{2R} = \frac{E^2\, R}{R^2 + (X_L - X_C)^2}$$



$$R^2 = (X_L - X_C)^2$$

And tanking the square root of both side

$$\pm R = X_L - X_C$$

$$\pm R = \left(\omega L + \frac{1}{\omega C}\right)$$

$$\left(\frac{\omega^2 LC - 1}{\omega C}\right) = \pm R$$

$$\omega^2 LC - 1 = \pm \omega RC \;\; (at\ half\ power)$$

we see that the two half-power points occur on both sides of the resonant angular frequency, ωs.

When ω < ω_S, the term $\omega^2 LC$ must be less than 1. In this case the solution is determined as follows

$$\omega^2 LC - 1 = -\omega RC$$

$$\omega^2 LC + \omega RC - 1 = 0$$

$$\omega_1 = \frac{-RC + \sqrt{(RC)^2 + 4LC}}{2LC}$$

$$\omega_1 = \frac{-R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

In a similar manner, for ω > ωs, the upper half-power frequency is

$$\omega_2 = \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

Notice that ω_1 and ω_2 are in general not symmetrical around the resonant frequency ω_s, because the frequency response is not generally symmetrical. However, as will be explained shortly, symmetry of the half-power frequency around the resonant frequency is often a reasonable approximate.

The bandwidth (BW)of the circuit as

$$BW = \omega_2 - \omega_1$$

which gives,

$$BW = \frac{R}{L} \quad (\frac{rad}{s})$$

If the above expression is multiplied by ωs/ωs we obtain

$$BW = \frac{\omega_S R}{\omega_S L}$$

Since $Q_S = \frac{\omega_S L}{R}$ we further simplify the bandwidth as

$$BW = \frac{\omega_S}{Q_S} \qquad (\frac{rad}{s})$$

Because the bandwidth may alternately be expressed in hertz, the above expression is equivalent to having

$$BW = \frac{R}{2\pi L} = \left(\frac{f_S}{\omega_S}\right)\left(\frac{\omega_S}{Q_S}\right) = \frac{f_s}{Q_S} \qquad (Hz)$$

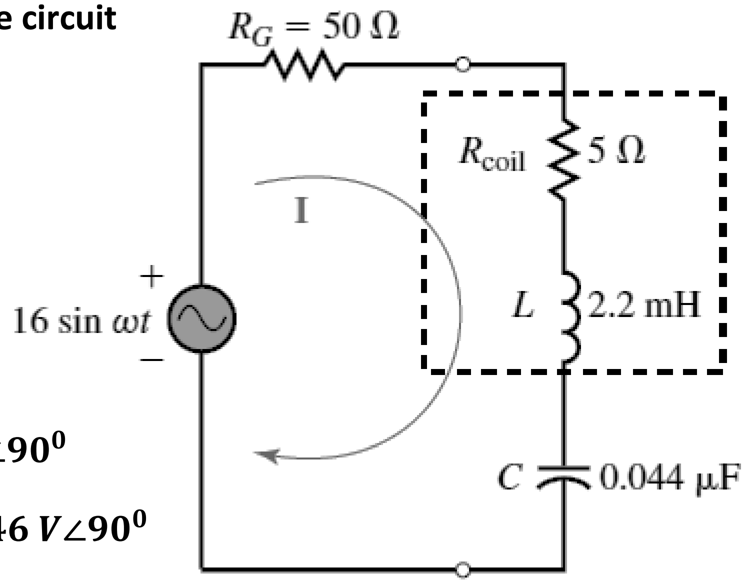
For high-Q circuit (Q>10) the half-power frequencies are, for all practical purposes, symmetrical around the resonant frequency and can be approximated as.

$$\omega_1 \approx \omega_S - \frac{BW}{2} \qquad , \omega_2 \approx \omega_S + \frac{BW}{2}$$

High –Q circuits are used often in commination networks

<u>Example:</u> For the circuit as shown below , Find the resonant frequency expressed as ω (rad/s) and f(Hz).

a) **Determine the total impedance at resonance.**
b) **Solve for I, V_L, and V_C at resonance.**
c) **Calculate reactive powers Q_C and Q_L at resonance.**
d) **Find the quality factor, Qs, of the circuit**



a) $\omega_S = \frac{1}{\sqrt{LC}} = 102Krad/s$

$$f = \frac{\omega_s}{2\pi} = 16.2KHz$$

$$Z = R = 55\ \Omega\angle 0^0$$

b) $I = \frac{E}{Z} = 0.206\ A\angle 0^0$

$$X_L = \omega L = j223.81\ \Omega = 223.81\ \Omega\angle 90^0$$

$$V_L = (0.206\angle 0^0)(223.81\angle 90^0) = 46\ V\angle 90^0$$

$$V_C = (0.206\angle 0^0)(223.81\angle -90^0) = 46\ V\angle -90^0$$

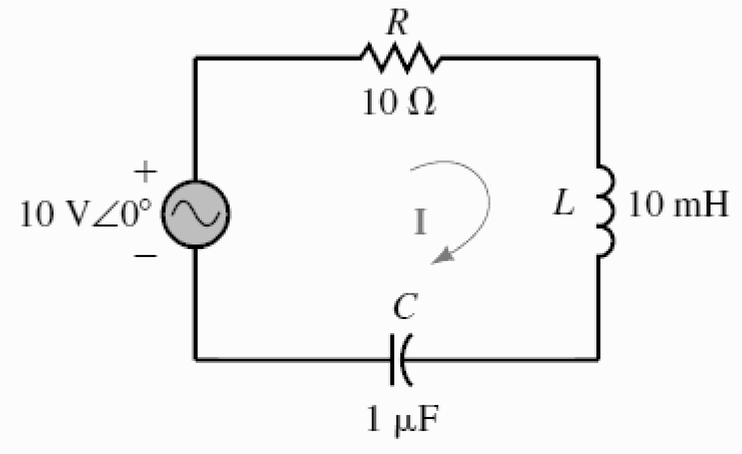
Notice that the voltage across the reactive elements is greater than the applied signal voltage. Although we use the symbol Q to designate both reactive power and the quality factor, the context of the question generally provides us with a clue as to which meaning to use.

c) $Q_L = (0.206)^2(223.81) = 9.49\ VAR$

$$Q_C = (0.206)^2(223.81) = 9.49\ VAR$$

d) $Q_S = \frac{X_L}{R} = 4.07$

Example: For the circuit shown



a) Determine the maximum power dissipated at resonance.
b) Determine the bandwidth of the resonant circuit and to arrive at the approximate half-power frequencies.
c) Calculate the actual half-power frequencies, ω1 and ω2, from the given component values.
d) Solve for the circuit current, I, and power dissipated at the lower half power frequency, ω1, found in Part (c).

Sol\a) $P_{max.} = \frac{E^2}{R} = 10\ W$

b) $\omega_s = 10\ Krad/s$

$$Q_S = \frac{X_L}{R} = 10$$

$$BW = \frac{\omega_S}{Q_S} = 1Krad/s$$

$$\omega_1 = \omega_S - \frac{BW}{2} = 9.5\frac{Krad}{s} \quad \rightarrow \quad \omega_2 = \omega_S + \frac{BW}{2} = 10.5\ Krad/s$$

c)

$$\omega_1 = \frac{-R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} = 9512.29\ \frac{rad}{s}, \qquad f_1 = 1514Hz$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} = 10512.49\frac{rad}{s}, \qquad f_2 = 1673.1Hz$$

Notice that the actual half-power frequencies are very nearly equal to the approximate values. For this reason, if(Q>10) it is often sufficient to calculate the cutoff frequencies by using the approximate.

d) At ω_1=9.51249 Krad/s

$X_L = \omega L = 95.12\Omega$

$X_C = 105.12\Omega$

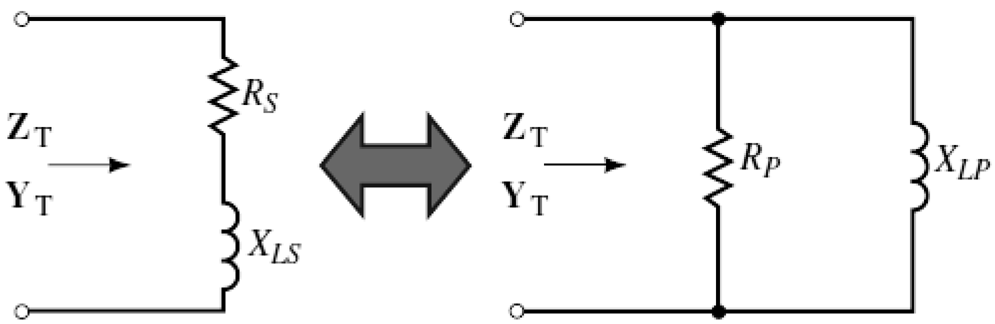
$Z = (10 - j10)\Omega$

$I = 0.707\ A\angle 45^0$

$P = I^2 R = 5W$

<u>Series-to-Parallel RL and RC Conversion:</u>

As we have already seen, an inductor will always have some series resistance due to the length of wire used in the coil winding. Even though the resistance of the wire is generally small in comparison with the reactance's in the circuit, this resistance may occasionally contribute tremendously to the overall circuit response of a parallel resonant circuit. We begin by converting the series RL network as shown in Figure below



The networks of Figure can be equivalent only if they each have the same input impedance, Z_T (and also the same input admittance, Y_T).

The input impedance of the series network of Figure is given as

$Z_T = R_S + jX_{LS}$

which gives the input admittance as

$$Y_T = \frac{1}{R_S + jX_{LS}}$$

Multiplying numerator and denominator by the complex conjugate, we have,

$$Y_T = \frac{R_S - jX_{LS}}{R_S^2 + X_{LS}^2}$$

$$Y_T = \frac{R_S}{R_S^2 + X_{LS}^2} - j\frac{X_{LS}}{R_S^2 + X_{LS}^2}$$

we see that the input admittance of the parallel network must be

$$Y_T = G_P - jB_{LP}$$

which may also be written as

$$Y_T = \frac{1}{R_P} - j\frac{1}{X_{LP}}$$

The following equations enable us to convert a series RL network into its equivalent parallel network:

$$R_P = \frac{R_S^2 + X_{LS}^2}{R_S} \dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$X_{LP} = \frac{R_S^2 + X_{LS}^2}{X_{LS}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . (2)$$

If we were given a parallel RL network, it is possible to show that the conversion to an equivalent series network is accomplished by applying the following equations

$$R_S = \frac{R_P X_{LP}^2}{R_P^2 + X_{LP}^2} \dots\dots\dots\dots\dots\dots\dots\dots (3)$$

$$X_{LS} = \frac{R_P^2 X_{LP}}{R_P^2 + X_{LP}^2} \dots\dots\dots\dots\dots\dots\dots\dots\dots .. (4)$$

Equations (1) to (2) may be simplified by using the quality factor of the coil. Multiplying Equation 1 by RS/RS, we have

$$R_P = R_S \frac{R_S^2 + X_{LS}^2}{R_S^2}$$

$$R_P = R_S(1 + Q^2)$$

Similarly, Equation 2 is simplified as

$$X_{LP} = X_{LS} \frac{R_S^2 + X_{LS}^2}{X_{LS}^2}$$

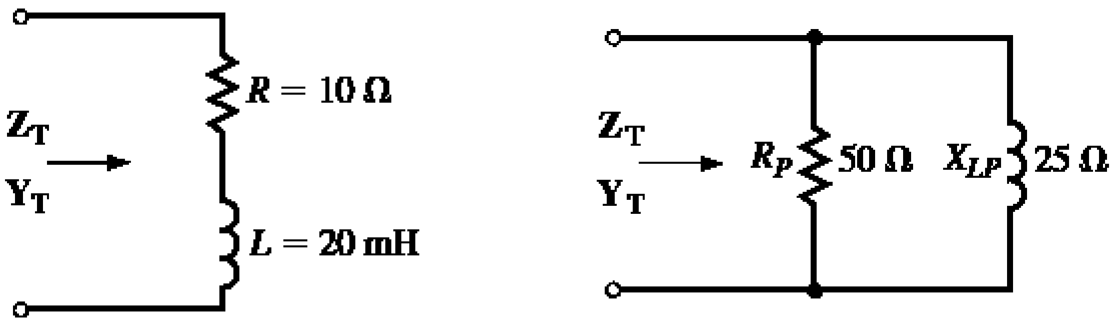
$$X_{LP} = X_{LS}(1 + \frac{1}{Q^2})$$

The inductive reactances of the series and parallel networks are approximately equal. Hence

$$R_P \cong Q^2 R_S \qquad (Q \geq 10)$$

$$X_{LP} \cong X_{LS} \qquad (Q \geq 10)$$

<u>Example</u>: For the series network of Figure, find the Q of the coil at ω=1000 rad/s and convert the series RL network into its equivalent parallel network. Repeat the above steps for ω= 10 krad/s.



sol/ At ω=1000 rad/s

$$X_L = \omega L = 20\Omega$$

$$Q = \frac{X_{LS}}{R_S} = 2$$

$$R_P = \frac{R_S^2 + X_{LS}^2}{R_S} = R_S(1 + Q^2) = 50\Omega$$

$$X_{LP} = \frac{R_S^2 + X_{LS}^2}{X_{LS}} = X_{LS}\left(1 + \frac{1}{Q^2}\right) = 25\Omega$$

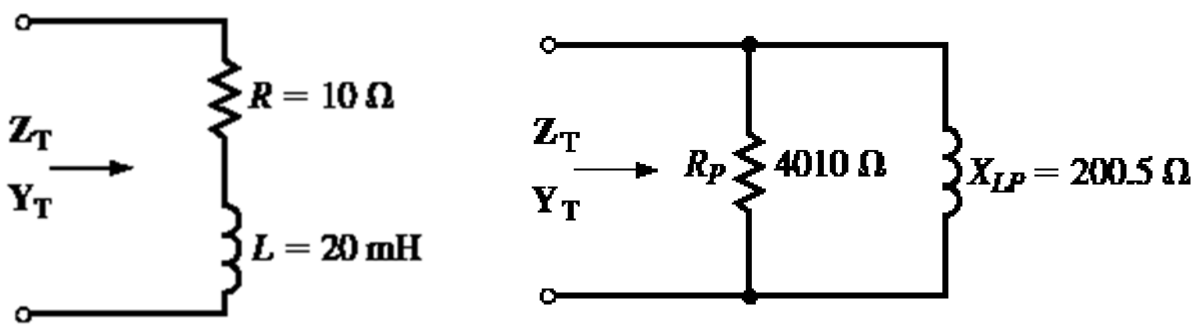
At ω= 10 krad/s

$$X_L = \omega L = 200\Omega$$

$$Q = \frac{X_{LS}}{R_S} = 200\Omega$$

$$R_P = R_S(1 + Q^2) = 4010\Omega$$

$$X_{LP} = X_{LS}\left(1 + \frac{1}{Q^2}\right) = 200.5\Omega$$



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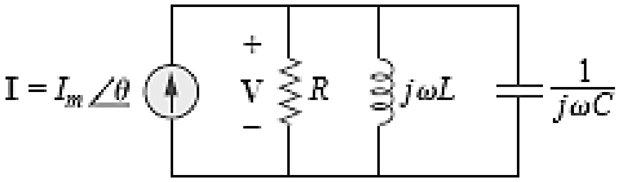
Parallel Resonance:

The parallel resonant circuit is best analyzed using a constant-current source, unlike the series resonant circuit which used a constant-voltage source.



Consider the LC "tank" circuit as shown. The tank circuit consists of a capacitor in parallel with an inductor. Due to its high Q and frequency response, the tank circuit is used extensively in communications equipment such as AM, FM, and television transmitters and receivers.

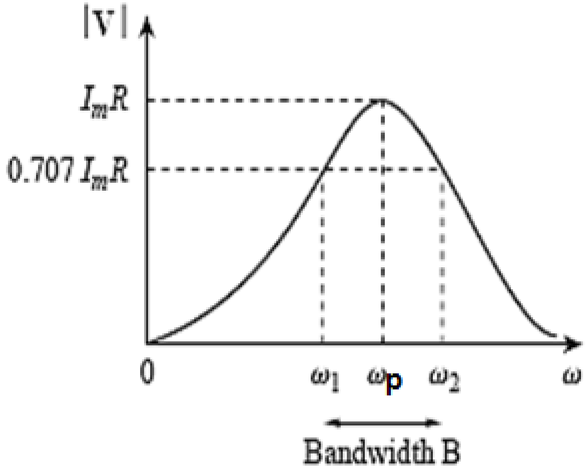
The input admittance of this network is

$$Y = \frac{I}{V} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$Y = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

Resonance occurs, When the

imaginary part of Y is zero,

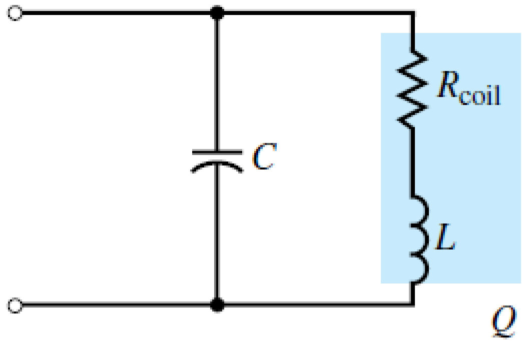


$$\left(\omega C - \frac{1}{\omega L}\right) = 0$$

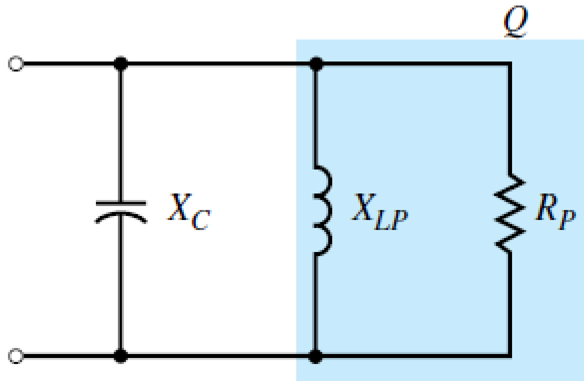
$$\omega_p = \frac{1}{\sqrt{LC}} \qquad (\frac{rad}{s})$$

Which is the same for the series resonant circuit. Notice that at resonance, the parallel LC combination acts like an open circuit, so that the entire current flows through R. Also, the inductor and capacitor current can be much more than the source current at resonance. The input impedance of this network at resonance is therefore purely resistive and given $Z_T = R_P$.

The circuit of Figure is not exactly a parallel resonant circuit(LRC), since the resistance of the coil is in series with the inductance.



In order to determine the frequency at which the circuit is purely resistive, we must first convert the series combination of resistance and inductance into an equivalent parallel network. The resulting circuit is shown in Figure



We determine the resonant frequency of a tank circuit by first letting the reactance's of the equivalent parallel circuit be equal:

$$X_{LP} = \frac{(R_{Coil})^2 + (X_L)^2}{X_{LS}} \qquad , R_P = \frac{R_S^2 + X_{LS}^2}{R_S}$$

$$X_C = X_{LP}$$

$$\frac{1}{\omega C} = \frac{(R_{Coil})^2 + (\omega L)^2}{\omega L} \qquad \rightarrow \frac{L}{C} = (R_{Coil})^2 + (\omega L)^2$$

$$\omega_p = \sqrt{\frac{1}{LC} - \frac{(R_{Coil})^2}{L^2}}$$

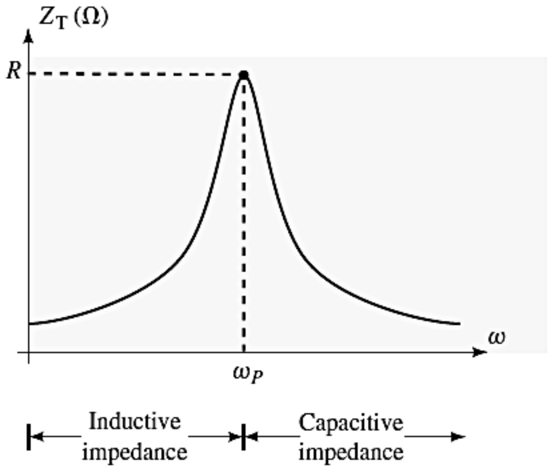
$$\omega_p = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{(R_{Coil})^2 C}{L}}$$

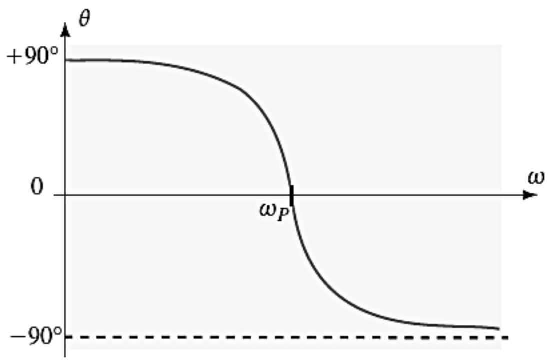
$$\omega_p \cong \frac{1}{\sqrt{LC}} \qquad \left(For\ \frac{L}{C} \gg 100R_{Coil}\right)$$

The inductor and capacitor reactance's cancel, resulting in a circuit voltage simply determined by Ohm's law as

$$V = IR\angle 0^0$$

The frequency response of the impedance of the parallel circuit is shown in Figure





Notice that the impedance of the entire circuit is maximum at resonance and minimum at the boundary conditions $(\omega = 0\frac{rad}{sec}\ and\ \omega \to \infty)$. This result is exactly opposite to that observed in series resonant circuits which have minimum impedance at resonance.

The Q of the parallel circuit is determined from the definition as

$$Q_P = \frac{recative\ power}{active\ power}$$

$$Q_P = \frac{(\frac{V^2}{X_L})}{(\frac{V^2}{R_L})} = \frac{R}{X_L} = \frac{R}{X_C}$$

This is precisely the same result as that obtained when we converted an RL series network into its equivalent parallel network.

$$I_R = \frac{V}{R} = I$$

$$I_L = \frac{V}{X_L\angle 90^0} = Q_P I\angle -90^0$$

$$I_C = \frac{V}{X_C\angle -90^0} = Q_P I\angle 90^0$$

At resonance, the currents through the inductor and the capacitor have the same magnitudes but are 180° out of phase. Notice that the magnitude of current in the reactive elements at resonance is Q times greater than the applied source current. Because the Q of a parallel circuit may be very large, we see the importance of choosing elements that are able to handle the expected currents.

In a manner similar to that used in determining the bandwidth of a series resonant circuit, it may be shown that the half-power frequencies of a parallel resonant circuit are

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \qquad (\frac{rad}{\sec})$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \qquad (\frac{rad}{\sec})$$

The bandwidth is therefore

$$BW = \omega_2 - \omega_1 = \frac{1}{RC} \qquad \left(\frac{rad}{sec}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

If $Q \geq 10$, then the selectivity curve is very nearly symmetrical around ω_p, resulting in half-power frequencies which are located at

$$\omega_1 \cong \omega_p \pm \frac{BW}{2}$$

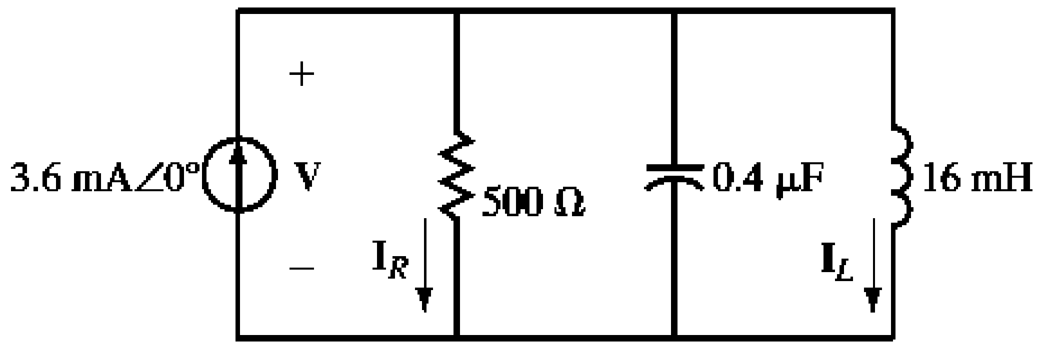
Multiplying Equation (1) by $(\frac{\omega_p}{\omega_p})$ results in the following:

$$BW = \frac{X_C \omega_p}{R}$$

$$BW = \frac{\omega_p}{Q_P}$$

Notice that Equation is the same for both series and parallel resonant circuits.

Example: Consider the circuit shown below



a) Determine the resonant frequencies, ω_o(rad/s) and f_o (Hz) of the tank circuit.
b) Find the Q of the circuit at resonance.
c) Calculate the voltage across the circuit at resonance.
d) For currents through the inductor and the resistor at resonance.
e) Determine the bandwidth of the circuit in both radians per second and hertz.
f) Sketch the voltage response of the circuit, showing the voltage at the half power frequencies.
g) Sketch the selectivity curve of the circuit showing P(watts) versus ω(rad/s).

Sol/ a)

$$\omega_p = \frac{1}{\sqrt{LC}} = 12.5\ Krad/s$$

$$f_p = \frac{\omega_p}{2\pi} = 1989\ Hz$$

b)

$$Q_p = \frac{R_P}{\omega L} = 2.5$$

C) At resonance V_C=V_L=V_R and so

$$V = IR = (3.6\ mA\angle 0^0)(500\angle 0^0) = 1.8V\angle 0$$

d)

$$I_L = \frac{V_L}{Z_L} = 9mA\angle -90^0$$

$$I_R = I = 3.6\ mA\angle 0$$

e) $BW\left(\frac{rad}{s}\right) = \frac{\omega_P}{Q_P} = 5\ Krad/s$

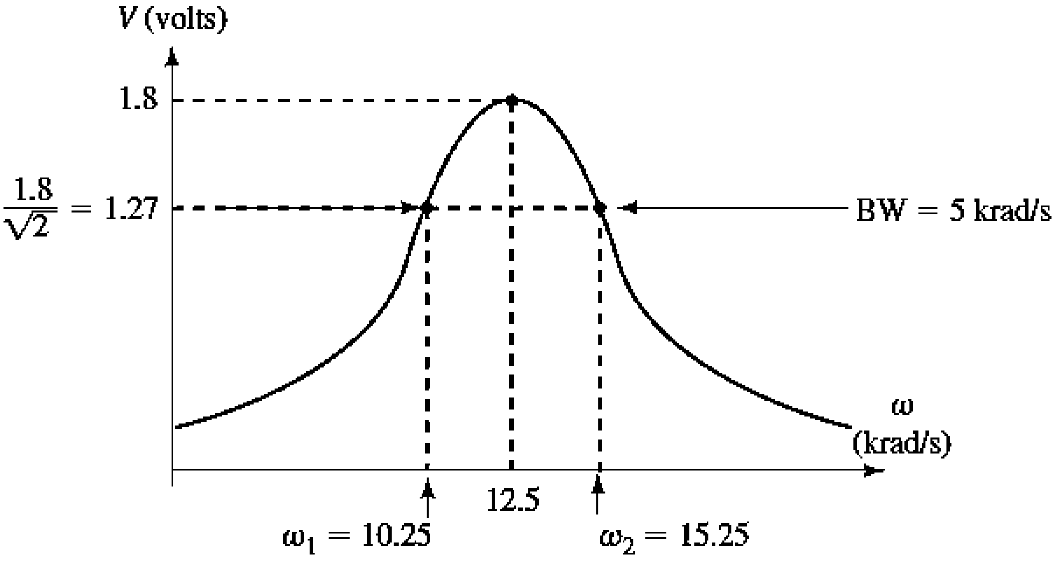
$$BW(Hz) = \frac{BW(\frac{rad}{s})}{2\pi} = 795.8\ Hz$$

f) Since the Q of the circuit is less than 10, The half-power frequencies are:

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad = 10248\ \left(\frac{rad}{sec}\right)$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} = 15248 \quad \left(\frac{rad}{sec}\right)$$

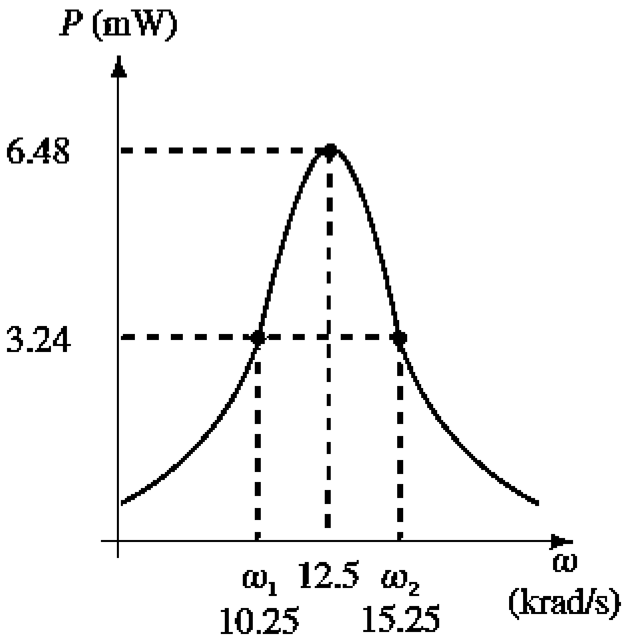
The resulting voltage response curve is illustrated in Figure



g) The power dissipated by the circuit at resonance is

$$P = \frac{V^2}{R} = 6.48\ mW$$

The selectivity curve is now easily sketched as shown in Figure below



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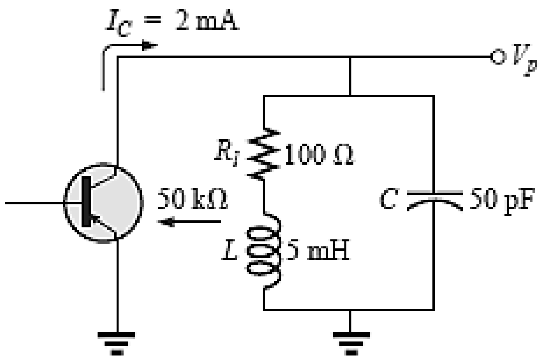
EXAMPLE : **The equivalent network for the transistor configuration of Figure below.**



a. Find, ω_p, f_p.
b. Determine Q_p.
c. Calculate the BW.
d. Determine V_p at resonance.
e. Sketch the curve of V_C versus frequency.

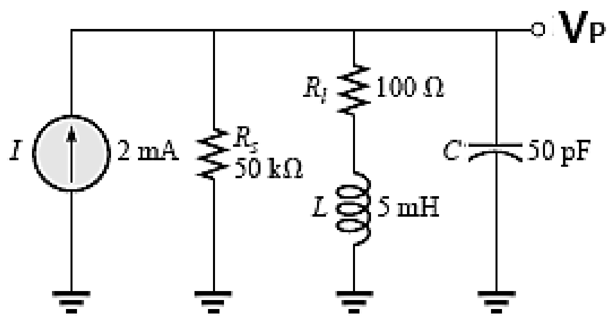
Sol/ a) since $\frac{L}{C} \geq 100\, R_{Coil}$

Then , $\omega_p \cong \frac{1}{\sqrt{LC}} = 2000Krad/s$

$f_p = \frac{\omega_p}{2\pi} = 318.471KHz$

X_L =10KΩ

$Q_l = \frac{X_L}{R} = 100$



b) Since $Q_l \geq 10$

$R_P \cong Q^2 R_S = 1M\Omega$

$R_{eq} = R_S \parallel R_P = 47.619K\Omega$

$Q_P = \frac{R_{eq}}{X_L} = 4.76$

Note the drop in Q from $Q_l = 100$ **to** $Q_P = 4.76$ **due to R_S**

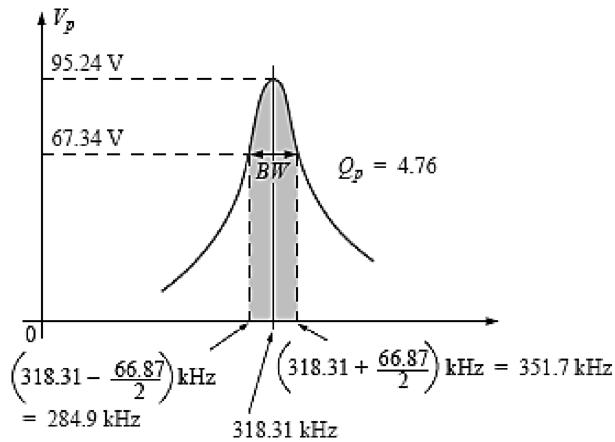
C) $BW = \frac{\omega_p}{Q_P} = 420.168\, Krad/s$

$BW = 66.9\, KHz$

d) $V_P = IZ_P = 95.24\, V$

e) $f_1 = f_P - \frac{BW}{2} \cong 284.9KHz$

$f_2 = f_P + \frac{BW}{2} \cong 351.7KHz$



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