Bevel Gears

Bevel gears are cut on conical blanks to be used to transmit motion between intersecting shafts. The simplest bevel gear type is the straight-tooth bevel gear or straight bevel gear as can be seen from Figure 1. As the name implies, the teeth are cut straight, parallel to the cone axis, like spur gears.

Fig.(1) Bevel gears

Straight Bevel Gears

Straight bevel gears are the most economical of the various bevel gear types. These gears are used primarily for relatively low-speed applications with pitch-line velocities up to 1000 fpm, where smoothness and quietness are not significant considerations. However, with the use of a finishing operation (e.g., grinding), higher speeds have been successfully handled by straight bevel gears.

Geometry

The geometry of bevel gears is shown in Figure (2).The size and shape of the teeth is defined at the large end on the back cones. They are similar to those of spur gear teeth. Standard straight bevel gears are cut by using a 20° pressure angle and full-depth teeth, which increase the contact ratio and the strength of the pinion.

The diametral pitch refers to the back- cone of the gear. Therefore, the relationships between the geometric quantities and the speed for bevel gears are given as follows:

$$d_p = \frac{N_p}{P}, \quad d_g = \frac{N_g}{P}$$

$$tan\alpha_p = \frac{N_p}{N_g}, \quad tan\alpha_g = \frac{N_g}{N_p}$$

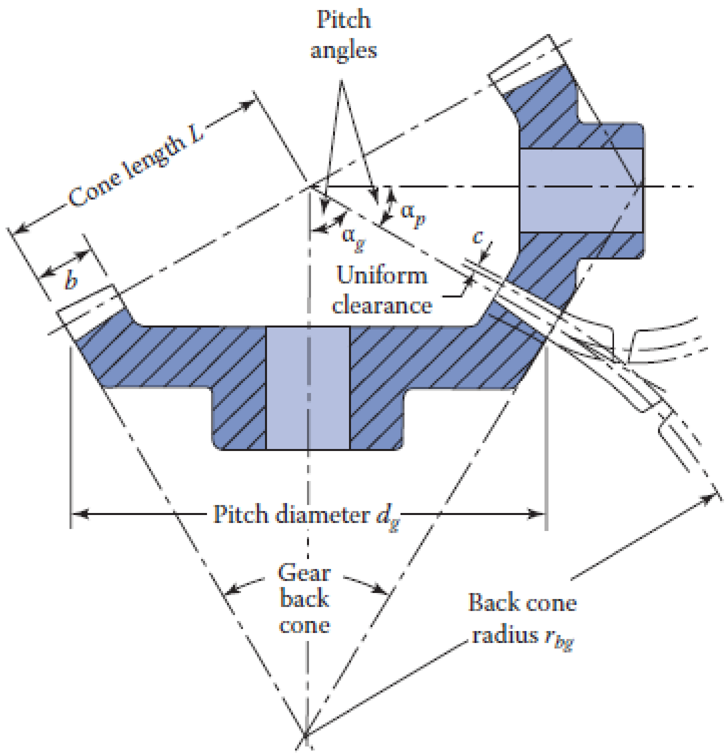


Fig.(2) Bevel gear .

$$r_s = \frac{\omega_g}{\omega_p} = \frac{N_p}{N_g} = \frac{d_p}{d_g} = tan\alpha_p = cot\alpha_g$$

where

d = the pitch diameter

P = the diametral pitch

N = the number of tooth

α = the pitch angle

ω = the angular speed

r_s = the speed ratio.

It is to be noted that, for 20° pressure angle straight bevel gear teeth, the face width (b) should be made equal to:

$$b = \frac{L}{3} \quad or \;\; b = \frac{10}{P}$$

whichever is smaller, The uniform clearance is given by the following formula:

$$c = \frac{0.188}{P} + 0.002\ in$$

The quantities L and c represent the pitch cone length and clearance, respectively (Figure 2).

Virtual Number of Teeth:

The virtual number of hypothetical spur gear can be calculated from the following equation:

$$N'_p = 2r_{bp}P, \;\; N'_g = 2r_{bg}P$$

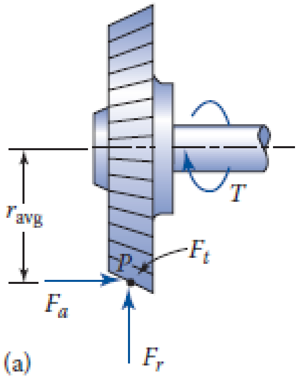
This may be written in the following convenient form:

$$N'_p = \frac{N_p}{cos\alpha_p} \quad , N'_g = \frac{N_g}{cos\alpha_g}$$

in which (r_b) is the back cone radius and (N) represents the actual number of teeth of bevel gear.

Tooth Loads of Straight Bevel Gears

The forces at the midpoint of the bevel gear tooth can be shown in figure(3).



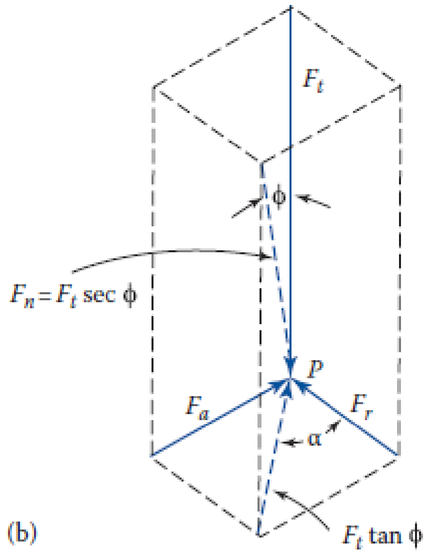


Fig.(3) forcer at the midpoint of bevel gear

The transmitted tangential load or tangential component of the applied force, acting at the pitch point P, is then

$$F_t = \frac{T}{r_{avg}}$$

Here

T represents the torque applied

r_{avg} is the average pitch radius of the gear under consideration.

The resultant force normal to the tooth surface at point P of the gear has value

$$F_n = F_t \sec\varphi$$

(Figure 3.b). The projection of this force in the axial plane, $F_t \tan\varphi$, is divided into the axial and radial components

$$F_a = F_t \tan\emptyset \sin\alpha$$

$$F_r = F_t \tan\emptyset \cos\alpha$$

where

F_t = the tangential force

F_a = the axial force

F_r = the radial force

φ = the pressure angle

α = the pitch angle

Ex:

A set of 20° pressure angle straight bevel gears is to be used to transmit 20 hp from a pinion operating at 500 rpm to a gear mounted on a shaft that intersects the shaft at an angle of 90° (Figure 4). Calculate

a. The pitch angles and average radii for the gears

b. The forces on the gears

c. The torque produced about the gear shaft axis

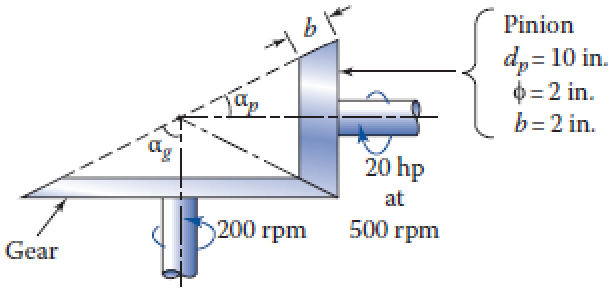


Fig.(4)

Solution

$$\boldsymbol{r_s = \frac{n_g}{n_p} = \frac{200}{500} = \frac{1}{2.5} = \frac{d_p}{d_g}}$$

Or

$$\boldsymbol{d_g = 2.5 d_p}$$

$$\alpha_p = tan^{-1}\left(\frac{1}{2.5}\right) = 21.8^o$$

$$\alpha_g = 90 - \alpha_p = 90 - 21.8 = 68.2^o$$

$$r_{gavg} = r_g - \frac{b}{2} sin\alpha_g = 12.5 - 1sin68.2 = 11.6in$$

$$r_{pavg} = r_p - \frac{b}{2} sin\alpha_p = 5 - 1sin21.8 = 4.6in$$

b. Through the use of the power equation

$$\mathrm{hp} = \frac{\mathrm{F_t V}}{33000}$$

$$\mathrm{F_t} = \frac{33000\mathrm{hp}}{\mathrm{V}}$$

$$\mathrm{V} = \frac{\pi \mathrm{d_{pavg} n_p}}{12} = \frac{\pi(9.2)500}{12} = 1204.2\mathrm{fpm}$$

$$\mathrm{F_t} = \frac{33000(20)}{1204.2} = 548\mathrm{lb}$$

$$F_a = F_t tan\emptyset sin\alpha_p = 548(tan20)(sin21.8) = 74lb$$

$$F_r = F_t tan\emptyset\ cos\alpha_p = 548(tan20)(cos21.8) = 185lb$$

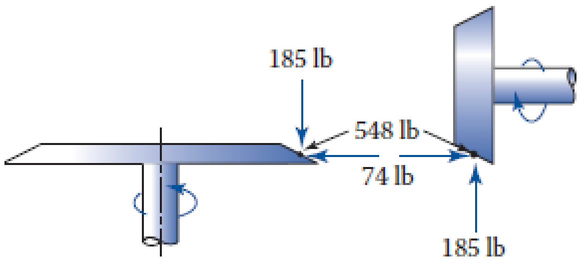


Fig.(5) Forces on bevel gear and pinion

c. $T = F_t\left(\frac{d_{gav}}{2}\right) = 548(11.6) = 6.356 kips.in$

Bevel Gear Tooth Bending and Wear Strengths

The allowable bending load is given by:

$$F_b = \frac{\sigma_o bY}{K_f P}$$

The factor Y for a gear of N' virtual number of teeth.

Buckingham Equation:

Due to the difficulty in achieving a bearing along the entire face width *b*, about three quarters of *b* alone is considered as effective. So the allowable *wear load* can be expressed as:

$$F_w = \frac{0.75 d_p bKQ'}{cos\alpha_p}$$

Where

d_p = the diameter measured at the back of the tooth

N' = the virtual tooth number

α = the pitch angle

K = the wear load factor.

For the satisfactory operation of the bevel gear sets, the usual requirement is that

$$F_b \geq F_d$$

and

$$F_w \geq F_d$$

(Q)A pair of bevel gears is to transmit 15 hp at 500 rpm with a speed ratio of (0.5). The 20 pressure angle pinion has an 8 in. back cone pitch diameter, 2.5 in. face width, and a diametral pitch of 7 teeth/in. Calculate the axial and radial forces acting on each gear. If the gears are made of SAE 1020 steel (WQ&T), will they be satisfactory from a bending point of view? Employ the Lewis equation and $K_f = 1.4$.

Solution:

$$r_s = \frac{d_p}{d_g}$$

$$\therefore d_g = \frac{d_p}{0.5} = \frac{8}{0.5} = 16in$$

$$r_s = tan\alpha_p$$

Or

$$\alpha_p = tan^{-1}r_s = tan^{-1}0.5 = 26.56^o$$

$$\alpha_g = 90 - 26.56 = 63.44^o$$

Hence

$$r_{gavg} = r_g - \frac{b}{2}sin\alpha_g = 8 - \frac{2.5}{2}sin63.44 = 6.88in$$

$$r_{p,avg} = r_p - \frac{b}{2}sin\alpha_p = 4 - \frac{2.5}{2}sin26.56 = 3.44in$$

$$\mathrm{F_t} = \frac{33000\mathrm{hp}}{\mathrm{V}}$$

$$\mathrm{V} = \frac{\pi \mathrm{d_{pavg}} \mathrm{n_p}}{12}$$

$$\mathrm{V} = \frac{\pi \times 6.88 \times 500}{12} = 900.58fpm$$

$$\mathrm{F_t} = \frac{33000 \times 15}{900.58} = 549.63lb$$

$$F_a = F_t tan\emptyset sin\alpha_p = 549.63(tan20)(sin26.56) = 88.4lb$$

$$F_r = F_t tan\emptyset\ cos\alpha_p = 549.63(tan20)(cos26.56) = 178.937lb$$

To discuss the suitability of the chosen material from the bending strength point of view:

$$F_b = \frac{\sigma_o bY}{K_f P}$$

The virtual number of teeth must be calculated as follows:

$$N_p{}' = 2r_{bp}P$$

$$N_p{}' = 2 \times 4 \times 7 = 56tooth$$

Take Lewis form factor for a gear with 50tooth

$$Y = 0.408$$

$$\sigma_o\ for\ material\ SAE\ 1020steel(WQand\ T) = 18ksi$$

$$F_b = \frac{18000 \times 2.5 \times 0.408}{1.4 \times 7} = 1873.46lb$$

Since

$$\mathrm{V} = 900.58fpm$$

$$F_d = \frac{600 + v}{600} F_t \qquad 0 < \ v \leq 2000fpm$$

Or

$$F_d = \frac{600 + 900.58}{600} \times 549.63 = 1374.6lb$$

Since $F_b \geq F_d$

The chosen material is suitable from the bending strength point of view.

(Q) A pair of 20° pressure angle bevel gears of N_1 = 30 and N_2 = 60 has a module m of 8.5 mm at the outside diameter. Determine the power capacity of the pair, using the Lewis and Buckingham equations.

The gears have face width of 70 mm, K_f = 1.5, and the pinion rotates at 720 rpm. The gears are made of steel SAE1040 and hardened to about 200 Bhn.

Solution

$$F_b = \frac{\sigma_o bYm}{K_f}$$

$$\sigma_o = 172MPa\ (from\ table)$$

To select the Lewis form factor the virtual number of teeth must be calculated:

$$N'_p = \frac{N_p}{cos\alpha_p}$$

$$r_s = \frac{d_p}{d_g} = \frac{30}{60} = 0.5$$

$$\alpha_p = tan^{-1}r_s = tan^{-1}0.5 = 26.56^o$$

$$\alpha_g = 90 - 26.56 = 63.44^o$$

$$N'_p = \frac{30}{cos26.56} = 33o.5\ tooth$$

Chose Y=0.358

$$F_b = \frac{172 \times 70 \times 0.358 \times 8}{1.5} = 22988.37N$$

$$F_w = \frac{0.75d_pbKQ'}{cos\alpha_p}$$

$$Q' = \frac{2N'_g}{N'_p + N'_g}$$

$$N'_g = \frac{N_g}{cos\alpha_g} = \frac{60}{cos63.43} = 134.14$$

$$Q' = \frac{2 \times 134.14}{33.5 + 134.14} = 1.6$$

To calculate the pitch circle diameter of the pinion can be evaluated as

$$d_p = mN_p = 3.5 \times 30 = 105mm$$

K=0.545MPa

$$F_w = \frac{0.75 \times 105 \times 70 \times 0.545 \times 1.6}{cos26.56} = 5374N$$

To calculate the dynamic load

$$r_{pav} = r_p - \frac{b}{2} sin\alpha_p = 52.5 - \frac{70}{2} sin26.56 = 36.85mm$$

$$V = \frac{\pi dn}{60} = \frac{\pi \times (0.036 \times 2) \times 720}{60} = \frac{2.71m}{s} \, or \; 537.4fpm$$

$$\therefore F_d = \frac{600 + V}{600} F_t = \frac{600 + 537.4}{600} = 1.89F_t$$

The value of the tangential force transmitted must be calculated according to the lowest value of F_b and F_w.

So that

$$F_w = 1.89F_t$$

$$5374 = 1.89F_t$$

Or $$F_t = 2834.88N$$

$$Power = F_t \times V = 2834.34 \times 2.714 = 7693.7W = 7.693kW$$